Original Article

On the Possibility That We Think in a Quantum Probabilistic Manner

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Abstract

My discussion is articulated under the neurological as well as the psychological profile. I insist in particular on the view that mental events arise in analogy with quantum probability fields. I review some results obtained on quantum cognition discussing in detail those that we obtained on quantum interference in mental states during perception-cognition in ambiguous figures. Frequently, I use the approach to quantum mechanics by Clifford algebra. I insist in particular on two recent results. The first is the justification that I obtain of the von Neumann postulate on quantum measurement and the second relates my Clifford demonstration on the logical origins of quantum mechanics and thus on the arising feature that quantum mechanics relates conceptual entities. The whole discussion aims me to support the conclusion that we think in a quantum probabilistic manner.

Key Words: quantum cognition, Clifford algebra, quantum probability fields, logical origins of quantum mechanics

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1. Introductory Remarks

With the advent of functional brain imaging technologies, neuroscience and neuro-psychology have reached satisfactory levels of understanding and knowledge. It is of great relevance that has been identified brain areas that are involved in a wide variety of brain functions including learning and memory. On the other hand, the genetic and biochemical approaches offer a constant contribution in this direction producing step by step new important advances under the profile of the investigation, research, and clinic application.

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These are very valuable studies providing knowledge on the functional role of different brain areas. However neuroscience finds it hard to identify the crucial link existing between empirical studies that are currently described in psychological terms and the data that arise instead as described in neuro-physiological, genetic, and biochemical applications. These studies continue every day with a methodological approach that is well clear. We have a kind of prevailing tendency in which we are inclined to assume that the measurable properties of the brain through functional imaging technology and biochemical discoveries, should be finally sufficient to achieve an appropriate explanation of the psychologically phenomenology that occurs during neuropsychological experiments.

I am a physicist that has some mental reservation on this approach. I am convinced that intrinsically mental and experiential functions such as "feeling" and "knowing" and the other basic psychological functions and attitudes cannot be described exclusively in terms of knowledge achieved by the previously mentioned approach. They require in addition an adequate physics in order to be actually explained. I am profoundly persuaded about such basic statement, and I will explain here the reasons of my firm belief. Let me start with some considerations.

We have some well established ideas about some foundations on the manner in which biological systems work. Biological matter, including the brain, is ubiquitously arranged by protein molecules. According to Monod, allosteric enzymes are logic elements. They interact in information-processing circuits whose very fundamental structure is essentially probabilistic. In textbooks neurons are described quite classically with their tree-like dendrites and spines functioning essentially as deterministic adding devices. They weight sums of synaptic inputs. However, neurophysiological research has often evidenced that such approach could be also profoundly approximated. A single neuron could be very distant from being a simple additive device. According to Crick and to Koch (Crick & Koch, 1998) it could be a kind of highly complex information-processing structure. Studies by artificial neural networks are revealing that in a general framework we could be in presence of a very large and highly complex stochastic-computational arrangement about which we know very little in detail.

Eccles and Beck in 1992 (Beck and Eccles, 1992; 2003; Eccles, 1990; Beck, 1996; Margenau, 1950; 1953; Wolf, 1989) obtained by direct calculations that the synapses in the cortex may respond in a probabilistic manner to neural excitation; a probability that, given the small dimensions of synapses, should be governed by quantum uncertainty. These authors produced direct estimations that still today result very convincing. The first detailed quantum model of quantum conjunction synapse was given by the physicist, Evan Walker (Walker, 1977). In 1970 he proposed a synaptic tunneling model in which electrons can "quantum tunnel" between adjacent neurons, thereby creating a virtual neural network overlapping the real one. It is this virtual nervous system that for Walker produces consciousness and that it can direct the behavior of the real nervous system. In short the real nervous system operates by means of synaptic messages while the virtual one operates by means of quantum tunneling. I think that we arrive to a similar conclusion adopting the view of Eccles and of Margenau. It is the abstract field of probabilities that in quantum mechanics determines events. In his 1992 article, Eccles offered plausible arguments for mental events causing neural events via the mechanism of wave function collapse. Conventional operations of the synapses depend on the operation of ultimate synaptic units called buttons. Eccles states that, these synaptic buttons, when excited by an all-or-nothing nerve impulse, deliver the total content of a single synaptic vesicle, not regularly, but probabilistically. In Eccles words;

> *"Excitation of synaptic buttons delivering the total content of a single synaptic vesicle represents the first intrinsically probabilistic event in the brain. Eccles studied in detail the problem, evidencing that a refined physiological analysis of the synapse shows that the effective structure of each button is a paracrystalline presynaptic vesicular grid with about 50 vesicles. The existence of such a crystalline structure is suggestive of quantum physical laws in operation."*

Eccles focused attention on these para-crystalline grids as the targets for non-material events. He discussed in detail how the probability field of quantum mechanics, which carries neither mass nor energy, can nevertheless be envisioned as exerting effective action at the microlevels of quantum events. In the event of a sudden change in the probability field brought on by the observation of a complementary observable, there would be a change in the probability of emission of one or more of the vesicles. The action of altering the probability field without changing the energy of the physical system involved can be found by the equation governing the Heisenberg principle of uncertainty.

To be clear: for cortical nerve terminals, the observed fraction of action potential pulses that result in exocytosis is considerably less than 100%. This can also be modeled classically, but the large Heisenberg uncertainty in the locations of the triggering calcium ions, entails that the classical uncertainties will carry over to similar quantum uncertainties**.** At this stage of elaboration some different authors suggested that the sudden change in the probability field resulting from an observation could be the mechanism by which mental events trigger neural events.

Eccles concluded that calculations based on the Heisenberg uncertainty principle show that the probabilistic emission of a vesicle from the paracrystalline presynaptic grid could conceivably be modified by mental intention in the same manner that mental intention modifies a quantum wave function.

For experimental evidence showing how mental events influence neural events, Eccles pointed to the papers by Roland *et al.*, (Roland *et al.,* 1980) who recorded, using radioactive *xenon*, the regional blood flow (rBF) over a cerebral hemisphere while the subject was making a complex pattern of finger-thumb movements. They discovered that any regional increase in rBF is a reliable indicator of an increased neuronal activity in that area. Another evidence, using the same technique of monitoring rBF, showed that silent thinking has an action on the cerebral cortex. For example, merely placing one's attention on a finger that was about to be touched, showed that there was an increase in rBF over the postcentral gyrus of the cerebral cortex as well as the mid-prefontal area. A lot of studies conducted in recent years by fMRI substantially indicate that this is the way.

Eccles concluded that non-material mental events in the brain are at individual microsites, the presynaptic vesicular grids of the buttons. Each button operates in a probabilistic manner in the release of a single vesicle in response to a presynaptic impulse. It is this probability field that Eccles believed to be influenced by mental action that is governed in the same way that a quantum probability field undergoes sudden change when as a result of observation the quantum wave function collapses. By this way we arrive to the following basic conclusion: mental events cause neural events analogously to the manner in which probability fields of quantum mechanics are causatively responsible for physical events. As well as Wolf outlines (Beck and Eccles, 1992; 2003; Eccles, 1990; Beck, 1996; Margenau, 1950; 1953; Wolf, 1989) For completeness we report here in Fig1 also the basic scheme as it was used, published, represented and discussed in detail by Eccles in all his papers on this matter

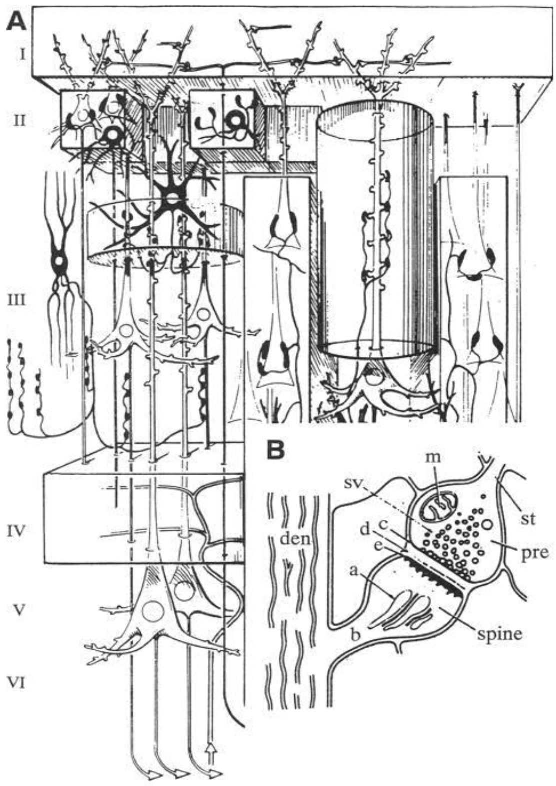


Figure 1. (A) Three-dimensional construct by J. SzentAgothai showing cortical neurons of various types. (B) Detailed structure of a spine (sp) synapse on a dendrite (den); st: Axon terminating in synaptic bouton or presynaptic terminal (pre); sv: synaptic vesicles; c: presynaptic vesicular grid; d: synaptic cleft; e: postsynaptic membrane; a: spine apparatus; b: spine stalk; m: mitochondrion. *Figure 1 has been taken Proc Natl Acad Sci U S A. 1992; 89 (23): 11357–11361*.

All these are important and fundamental elaborations. I have used terms as probability, link of quantum filed of probability and mental events, irreducible indeterminism. When a physicist starts to speak about structures that admit a so large number of abstract entities, he always feels to be panic-stricken. I am speaking about science or idealizations!

To be clear: all the scientific theories introduce mathematical models, and all they "approximate" or "idealize" in some manner our reality. When in such text I use terms as probability or quantum field of probabilities or "equivalence of probability with space of mental events", or still irreducible indeterminism, it is here that I see the risk of idealizations also if, generally speaking, the admissibility of idealizations in theorizing is and remains of main interest in science. To comment the previous conclusions by Walker, Eccles, Beck and Margenau, I certainly appreciate their highly fascinating content but on the other hand I must be care that some idealizations certainly avoid the risk to result so extreme as to be considered physically inadmissible. As a rule, we need unquestionable verifications to accept any thesis in science. Therefore let us go on step by step.

First. In order to take seriously in consideration the possibility that synaptic transmission is realized by quantum tunneling and thus by the foundations and the rules of quantum mechanics, we have to perform direct calculations. In detail we need to calculate the MEEP, (miniature **e**nd **p**late **p**otentials) – Frequency of vesicle release as it is obtained by quantum mechanics and to compare such obtained theoretical results with the existing experimental data. Only such comparison may indicate if we have an agreement or not between experimental and theoretical data and, in case, such positive result may orientate in some manner our thesis that quantum mechanics has a fundamental role.

In quantum tunneling model formulated by Walker (Walker, 1977) we have that an electron transfer is made between two macromolecules, proteins lying in the presynaptic dark projections of Gray and the postsynaptic density at the cleft. It is postulated that the charge transferred across the synapse results in raising the protein in the presynaptic dark projections of Gray at higher energy levels. As consequence, conformational changes in these molecules forming the vesicle gates in the cleft membrane alter their size determining the macrogates both to open and to eject a vesicle that is to say its contents realizing synaptic conjunction. If we have an electron bound in the molecule that is separated by the synaptic cleft from a second similar molecule, its energy E and the frequency ν by which the electron attempts to escape for realizing quantum tunneling are well known experimentally. The frequency $\hat{\nu}$ by which the electron makes quantum tunneling from the first to the second molecule realizing vesicle release results to be

$$\hat{\nu} = \nu P \qquad (1)$$

being P the probability of tunneling that will be function $P(V_o, V_1)$ with V_0 height of the potential barrier. For purpose of calculations we assume here it having the value of $0.118\, eV$ as it arises from the experimental data and V_1 being instead the presynaptic depolarization. The value, L, of the synaptic cleft is considered to be of 180 A^0 as it is obtained from experimental data.

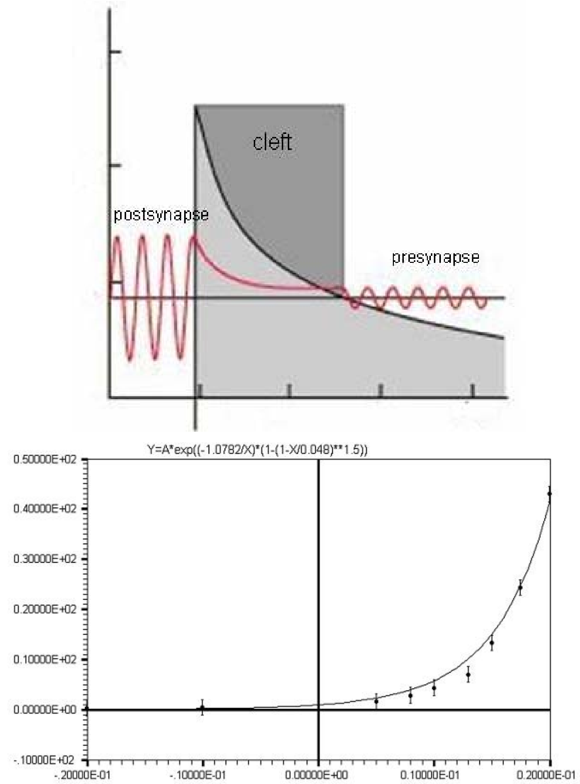


Figure 2. In ordinate we have the values of Mepp-frequency (frequency of vesicle release in sec^{-1}) and in abscissa those of depolarization potential. We have the theoretical curve as given by the (1) with the (3) and the experimental values, represented by data points, as they were obtained by Liley in 1956. Liley studied the appearance of MEPPs in isolated rat phrenic nerve-diaphram preparations. More recently experimental results were obtained also by H. von Gersdorff and coauthors (private communication) and they result very similar to those had by Liley time ago.

The performed calculations give the following expression for the potential barrier:

$$V(x)=(\frac{V_1-V_0}{L})x+V_0 \qquad 0\le x\le L \qquad \textbf{(2)}$$

and the value of probability quantum tunneling results after calculations to be

$$P(V_o,V_1)=\exp(-\alpha\frac{(V_0-E)^{3/2}}{V_1}(1-(1-\frac{V_1}{V_0-E})^{3/2})),$$

$$\alpha=\frac{4\sqrt{2m}}{3\hbar}L \qquad \textbf{(3)}$$

In (1) all the data are known and thus we may compare it with the experimental data of vesicle release for different values of the depolarization potential V_1.The results are given in Figure 2.

It may be observed that the agreement between theoretical and experimental data is excellent, ($R^2=0.992204$). I think that a more satisfying result cannot be asked in a comparison between experimental and theoretical data. So we may reach the first conclusion of the present text. I call it Evidence n.1.

Evidence n.1

It seems that we may conclude on the role of quantum mechanics in synaptic conjunction. Obviously it is only evidence. The agreement is excellent but is it sufficient such result to conclude the argument of synaptic conjunction? Certainly, this is not. I pose to myself the following questions and I think that the same neuroscientist will expect a precise indication about at least two other basic problems.

a) We have given here precise indications about the possible fundamental role of quantum mechanics in conjunction synapse between two adjacent neurons, but what is the counterpart of such conclusion for distant neurons? It is not the case that I insist here to outline the importance of such so fundamental question.

b) As basic rough scheme of functioning of a neuron we have the following scheme.

A neuron takes n-inputs i_j. A weight function is defined, and still a threshold ϑ and an output function f, and the complex mechanism may be summarized as it follows

$$f=\begin{matrix}1\ if\ \sum_{j=1}^{n}w_j i_j>\vartheta \\ -1\ otherwise\end{matrix}$$

In (2) we have given the expression of the acting potential barrier between the two adjacent neurons when the action potential arrives, but may we reconcile such expression with the time dependent mechanism that we have just evidenced by the output function f?

I attempt to answer to the first question. Science is based on the evidence of experimental results. I follow Walker in this argument. There is an experiment that was performed by Babich, AL. Jacobson, Bubash, and A. Jacobson in 1965. It was published on Science (Babich *et al.,* 1965). The title of the paper evidences immediately its content. It states: Transfer of a response to naïve rats by injection of ribonucleic acid extracted from trained rats. Rats were trained in a Skinner box to approach the food cup when a distinct click was sounded. Ribonucleic acid was extracted from the brains of these rats and injected into untrained rats. The untrained rats then manifested a significant tendency (as compared with controls) to approach the food cup when the click, unaccompanied by food, was presented. The conclusion seems to be: RNA serves to transfer information. It is well known that it does not affect the synaptic structures, but it is distributed throughout the brain. So, the explanation seems evident. RNA molecules develop the role of propagator molecules. Note that they live within few hundred angstroms of one another. Electron could use such propagator molecules as stepping ones, by subsequent tunneling into such propagator molecules they could develop the role to enable the electron to connect two distant synaptic molecules. Their lying within a few hundred angstroms, assures us that the quantum probability tunneling is not so drastically decreased by their presence. We have the potential barrier with the adjacent ones as due to such intermediate, propagator molecules and in this manner information may be transferred over large distances in the brain (Fig. 2a). We have here long-range quantum mechanical effects in the brain because we have to consider such succession of short range quantum effects in the time dependent formalism of quantum mechanics and the arising result is that we have a final wave function spreading out until it invades the entire space that it is allowed to it.

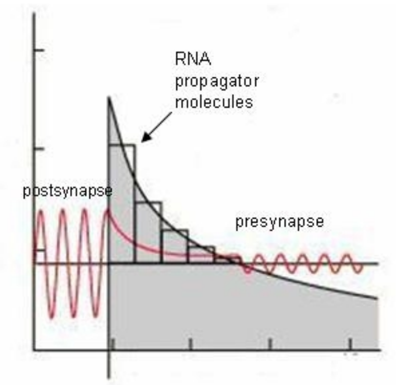


Figure 2a. Quantum tunneling at large distances by RNA propagator molecules

These considerations, here arranged in accord to Walker, seem to give a possible answer to the first posed question. However I have some basic comments. The first question is that, as well as I know, the experiment by the RNA never was repeated with success. But this may be a datum responding only to my insufficient documentation on this matter. The second comment is that under a probabilistic profile it does not seem so unreasonable to me the possibility of a step by step continued quantum tunneling. Let me do a digression on this matter.

We know that neural networks are retained the most promising models in cognitive neuroscience. In the classical approach the neural nets are retained as fully deterministic systems. The information processing of the neural nets bears the classic physics with basic determinism and locality. On the other hand there is an alternative approach in which the dynamics of the neural nets is modeled quantum mechanically and in this case it is assumed that a full quantum mechanical treatment is required. I am attempted to consider such two approaches as extreme idealizations. There is instead an alternative approach developed by Dugic and Rakovic (2000) in which it is proposed that the neural networks are classical physical systems but they are proposed at the same time to investigate quantum mechanical corrections to be added to the deterministic dynamics using quantum-mechanical tunneling. On the general plane this procedure recalls in some manner a new program of investigating what is usually called the border region between classical and quantum. These authors have studied in particular quantum corrections of an associative (attractor) neural nets. They adopt a very interesting strategy since they start from classical physics models of the associative neural nets and simultaneously they investigate quantum tunneling posing the question of minimizing the quantum fluctuations that are due to the tunneling.

They consider the physical states of the associative neural nets represented in the configuration (q-V) space. Here, each point of the horizontal axis represents a unique configuration, denoted by the vector **q.** In the vertical axis they represent the values of the potential energy of each configuration. The vector **q** having components $q_1, q_2, \ldots, q_n$ determines the state of the network as a whole while each q_i describes the state of each constituent neuron or, equivalently, of each synapse in the net Generally speaking, the (q-V) region has several potential wells that represent the patterns and, exhibiting the neural network quantum mechanical features, we may have tunneling between the patterns during a certain time interval τ. In brief we have as in physics a particle oscillating with a given frequency around the bottom of the well the moving is driven by external stimuli and results in the complex process of pattern formation and of pattern recognition.

The tunneling between the patterns is a tunneling between bound states; we have a similar effect to the case of macroscopic quantum tunneling in superconducting devices. For the formalization the authors use Pauli master equations. It may be assumed that only transitions between neighbor wells are effective and thus considering equations of the following form

$$\frac{dp_m}{dt} = W(p_{m+1} + p_{m-1} - 2p_m)$$

where the transitions $W_{mn} = W$ are assumed to be constant and the ratio $\tau / \tau_{tunneling}$ is considered, being τ the time in which the energy surface (q-V) remains unchanged and $\tau_{tunneling}$ that one of tunneling effect to occur. The p_m are the tunneling probabilities.

This completes our reasoning on possible repeated quantum tunneling. Let us go now back to our previous questions. There is still another comment. In (1) with (3) we calculated the probability of electron tunneling. Actually, the frequency of vesicle release f_{mepp} will depend on the number m giving the product of the actual number of electrons per molecule in the gate that must tunnel to determine conformational change and the number of molecule in the macrogate. Indicating by b the number of transmitting molecules in the postsynaptic elements for gate, and calling N the number of gates in the synapse, the f_{mepp} frequency of vesicle release is

$$f_{mepp} = \frac{Nb\hat{v}}{m}.$$

According to the calculations performed by Walker, the presynaptic membrane must have an electric charge $Q = CV_1$ sufficient to close the gates between firings of the synapse. Thus it must be $Q \approx mNe$ being e the electron charge. From existing experimental data we deduce that $Q \approx 646e$ as Walker calculates and $m \approx 9$. In conclusion we have a number of electrons involved. Also discharging the hypothesis of propagator molecules, it remains as more efficient the possibility of swapping quantum entanglement and of quantum entanglement as induced by noise. Let me give a very rough manner to explain quantum entanglement. Two particles are entangled when their states are correlated (generally exactly opposite) but both uncertain. To take a macroscopic example but only as general indication of quantum mechanics, imagine flipping a coin in a way where the flip was truly random, and covering it up before there was any observation of which side it landed on. In a quantum sense, the coin is literally both heads and tails until somebody looks - its wavefunction is a superposition of both states. Once somebody observes its state, the wavefunction collapses to one state or the other. Examples of quantum entanglement swapping may be given

Entanglement Swapping

In the last few years many results have been obtained on entanglement swapping. We may have entanglement swapping between two entangled pairs of particles (Des Brandes *et al.,* 2006; Bouda *et al.,* 2001; 2005). The theory shows that if a measurement is made simultaneously on elements (B) and (D) of the entangled pairs (A) (B) and (C) (D), the entanglement on pairs (A) (B) and (C) (D) collapses, but the elements (A) and (C) become entangled although they have never been in contact before. Other example. Entanglement swapping between two entangled gamma and two electrons. Let us admit two entangled gamma (0) and (1), interacting simultaneously with two electrons (2) and (3), as example in a crystal. We have the entanglement of the electrons (2) and (3) and the entanglement collapse between (0) and (1). These entangled electrons are then captured in the crystal traps and may stay as such in the traps for months or years at ambient temperature.

Still we have (Wu *et al.,* 2004) a different paradigm for entanglement generation: it is possible to entangle two particles that never interact directly by means of repeated measurements of a third subsystem that interacts with both. In addition to its conceptual interest, it is evident that this scheme offers practical advantages for long-range entanglement generation.

Finally, it has been shown that entanglement between two qubits can be generated if the two qubits interact with a common heat bath in thermal equilibrium, but do not interact directly with each other. In most situations the entanglement is created for a very short time after the interaction with the heat bath is switched on, but depending on system, coupling, and heat bath, the entanglement may persist for arbitrarily long times. This is an excellent mechanism that gives new light on the creation of entanglement (Braun, 2002).

Let us consider that brain contain basic noise. As example thermal excitations give origin to conformational change in molecules and thus cause spontaneously quantal release. The probability to find a site in an higher energy state is

$$p = e^{-E/kT}$$

and thus the probability that the previously (m) sites will open at a given instant of time is

$$p^m = e^{-mE/kT}$$

Scientists regularly discard up to 90 percent of the signals from monitoring of brain waves, one of the oldest techniques for observing changes in brain activity. They discard this data as noise because it produces a seemingly irregular patterns. The noisy activity of the brain at rest and in the background when we perform tasks, actually represents the majority of what the brain is really doing. The case is when we measure the cost of running the brain and find that this background activity accounts for most of it. There is pioneering research as major step forward in helping us in understanding how this background activity is organized.

Still We have Glial Cells in the Brain

When we consider the cells in the brain they are often too quick to overlook the most abundant type of brain cell. Scaffolding cells, otherwise known as glial cells, outnumber other types of neurons in the brain. Oligodendrocytes alone outnumber neurons by an estimated ratio of 3:1. There are four different types of glial cells – oligodendrocytes, astrocytes, microglia, and Schwann Cells – each with its own specific attributes, functions, and characteristics.

1. *Oligodendrocytes*

Oligodendrocytes are what many people consider the classic glial cell. This is the cell that forms a myelin sheath around axons. The myelin formed by these sheaths (or laminae) allows an action potential to propagate faster than if the axon were unmyelinated because of the time it takes for sodium channels to open. It takes longer for the channels to open than it does for the action potential to propagate, so a myelinated axon will transmit a signal at about 1.7 times the speed of an unmyelinated axon.

2. *Astrocytes*

Atrocities have many functions, some of which are unknown. Astrocytes have long processes, at the ends of which are footpods (or feet). These form tight junctions with one another and tend to surround blood vessels, forming the blood-brain barrier. Astrocytes also surround synaptic clefts, scavenging glutamate or other neurotransmitters that spill out of the synapse. Some people hypothesize that astrocytes even form their own network, feeding off the synapses of neurons like pyramidal cells.

3. *Microglia*

Microglia clear away dead or dying cells. These small glial cells can become activated and perform the function of a rudimentary immune system in the brain. Microglia are activated by trauma and help destroy infectious cells.

4. *Schwann Cells*

Schwann cells form myelin sheaths in the peripheral nervous system (more commonly known as the spinal cord). Unlike oligodendrocytes, each of which can myelinate more than one axons (in some cases up to 30 different axons), each Schwann cell only myelinates one axons. This completes our discussion on **Evidence n.1 .**

The second. We have still the more complex problem. It is that we relate the space of mental states with the field of probability as previously discussed. Is it an extreme idealization totally excluded from the possibility to be admitted in science? I answer as it is in my knowledge. We have given in the (2) the expression of the potential barrier when the action potential is arriving to the presynapse. It is obvious that it is a rough model that of course furnishes an excellent agreement with the experimental data. According to the classical scheme of neuron that we have previously illustrated, we are actually in presence of several inputs that are elaborated and combined by the neuron mechanism according to some different weight factors. Thus, in conclusion, the potential given in (2) is a rough, a mean approximation. We may refine it considering that the arriving of the n-inputs induces in the potential given in (2) some fluctuations that result time dependent. In conclusion, instead of considering the (2) as final expression of the potential barrier between the two synapse molecules, we must introduce instead a time-dependent potential tunneling containing random fluctuations.

In order to account for such feature we may as example write the (2) in the following time-dependent form

$$V(x,t) = k(t)V(x)$$

where we assume now that the barrier height, including $k(t)$, is now time-dependent. We may also consider other more detailed models in order to account for such random fluctuations in time. On this basis we may refine our calculations on quantum tunneling realizing synapse conjunction. There is a fundamental aspect that we have still to outline here. Quantum tunneling is an essential feature of quantum processes signed by irreducible indeterminism.

Let us consider a tunneling process. We have to connect it to a quantum wave function state that is the superposition of the two possible alternatives

$$\psi = c_1\psi_{tunneling-yes} + c_2\psi_{tunneling-not}$$

where $|c_1|^2$ represents the probability that the tunneling happens and $|c_2|^2$ the probability that it does not happen.

Now, in this quantum mechanical approach it is evident *that we have a profound link between the quantum probability* $|\psi|^2$ *and the weight factors* w_j *that are represented in the more classical scheme of the neuron given by*

$$f = \begin{matrix} 1 \text{ if } \sum_{j=1}^{n} w_j i_j > \vartheta \\ -1 \text{ otherwise} \end{matrix}$$

A Quantum Mechanical Model of Neuron

Of course the idea to connect the weight factors to the quantum wave function arises also in studies on artificial neuron networks with quantum mechanical properties (Ventura *et al.*, 1997). The basic idea here is that we have a direct link with the mechanism of synaptic conjunction explained in quantum mechanical terms on the basis of the previous arguments, and this gives a complete quantum mechanical model of the human neuron supported also from the previous experimental results.

In conclusion, the concept to relate the space of mental states with the filed of probability does not represent an extreme idealization.

However, by this way we reach an impasse. Quantum mechanics has a basic foundation. It is signed by an irreducible indeterminism. According to McIntyre (McIntyre, 2007), a paper written time ago on the general problem of "Thinking probabilistically", the arising problem is to explain how in our thinking sphere, the exquisite sense of mathematical precision, of unassailable mathematical truth, of the Platonic beauty and precision of simple mathematical curves and other deterministic constructs arise in our brain from the actions of our tens of billions of interconnected neurons subsisting in a immeasurable coexistence of fluctuations and of irreducible stochasticity and indetermination.

How is it that from a so high complex system, governed by stochasticity and by random fluctuations, by quantum intrinsic and irreducible indetermination, by a mental space conceived in some sense in an isomorphic field of probabilities and, more precisely, of quantum probability fields, it arises our thinking and our reasoning that results to be precise, austere, and deterministic?, just using McIntyre adjectiveness.

It is true. We are in accord with McIntyre paper. Our reasoning at the first stage uses an Aristotelian logic and its further developments. An unequivocal logic of thinking on which all we derive the uprightness of our thinking approach, arriving up to the abstract mathematical level of our formal advances.

Note that I have used two fundamental terms: determinism and Aristotelian logic. Let me comment about such two basic statements. By this way we will indicate if previously we used extreme idealizations or not.

The first is the determinism. Are we sure about determinism in our reality and in our thinking?

It is a celebrated statement that the classical dynamics of physics describes systems to be fully deterministic. It is still a paradigma that Nature exhibits determinism at the macroscopic level of description pertaining to classical physics. The governing equations of classical dynamics derive from Lagrange equations, from variational principles or from Newton's laws of motion. However, we must be careful in admitting determinism so at all. Determinism does not arise from this contextual physical framework. Our abstract mathematical level of reasoning and our precise thinking tendency to couple our statements with ordinary experience, lead directly to determinism. However, starting from this framework, often we do not sufficiently evidence or imprudently it is given silent that, in order to satisfy the requirement of determinism, we do not use only the previously mentioned physics. We add a posteriori a further relevant restriction. *We force to coexist the governing equations of classical dynamics and given initial conditions with an ad hoc added mathematical restriction. In order to obtain that our systems actually exhibit the claimed determinism, we impose to all such theoretical edifice of physics from the outside, that the differential equations describing a physical system, must satisfy the so called Lipschitz condition with the basic consequence that all the derivatives that we introduce at the mathematical and physical level, must be bounded.*[2]

We are used to admit reality going in a certain conceptual direction as it derives from our macroscopic experienced reasoning. Really, there is not another way for arising determinism. We admit it by an ad hoc assumption. It seems to respond more to and our kind of wishful thinking than other. And in fact we pay dear for such our tendency.

Let me give only one example. It is simple but very convincing. Take a particle in a one-dimensional motion decelerated by a friction force

$$F(v) = m\frac{dv}{dt} = -kv \qquad (4)$$

with m and v mass and velocity of the particle. Invoking the ad hoc assumption that F must satisfy Lipschitz condition and restriction we have the solution that is currently exposed in textbooks. We have

$$v = v_0 e^{-(k/m)t} \quad \text{with} \quad k = -(\frac{\partial F}{\partial v})_{v=0} > 0 \quad (v \to 0)$$

We unrealistically accept that the particle has $v \to 0$ for $t \to \infty$. We admit that the velocity of the particle goes to zero only after an infinite time. This is an abstraction in contrast with all that we actually observe with the experience. However, this is what we accepts and it results to be consolidated as arising from classical physics about such system. Really the matter does not go in this manner. We pay dear for such assumed Lipschitz condition, and we accept consequently that the particle approaches the equilibrium ($v = 0$) after an infinite time while instead really it approaches such physical condition in a finite time. Such physics describes an unreal situation.

Let us assume instead that the law of motion is

$$F = -kv - k_1 v^{\alpha}$$

with $k_1 << k$ and

$$\alpha = \frac{n}{n+2} \text{ with } n >> 1, \qquad (5)$$

being an odd number

For $\alpha \equiv 1$ you see that the two equations, the (4) and the (5), are very similar with the only exclusion of a small neighborhood of the equilibrium point ($v = 0$) with the fundamental difference that this time at this point the Lipschitz condition is violated. The little but substantial difference between our usual manner to solve this problem (the (4)) and the (5) gives enormous differences on the conceptual plane and on the plane of the Newtonian dynamical results. First of all, using the (5), correctly we have now that the time of approaching the equilibrium, $v = 0$, is finite as it must be. This time results to be

$$t_0 = \frac{m v_0^{1-\alpha}}{k_1(1-\alpha)}.$$

In addition, in a finite time the motion of the particle can reach the equilibrium and switch to the singular solution, and this switch is irreversible. The equilibrium point $v = 0$ of equation (5)

[2] It is such ad hoc mathematical restriction, that we choose to add from the outside that guarantees determinism in classical dynamics since it guarantees the uniqueness of the solutions of the used differential equations that are subjected to fixed initial conditions.

represents a terminal attractor which is infinitely stable and is intersected by all the attracted transients. The uniqueness of the solution at $\upsilon = 0$ is violated and also the motion for $t < t_0$ is totally forgotten. This is to say that we have irreversibility of the dynamics as required. The conclusion of this exposition is that if we let go out the Lipschitz restriction in some cases, we obtain a correct description of our reality.

So, is determinism an actual foundation or does it represent rather an expression of our obstinate wishful thinking? The results to let go out Lipschitz condition is not new here. It was introduced in literature by Michail Zak and Joseph P. Zbilut (Zak, 1997; Zak *et al.,* 1997; Zbilut, 2004). Systems violating Lipschitz conditions do not represent an abstract mathematical formulation. We have concrete cases of Lipschitz violation particularly in the sphere of the biological dynamics. With A. Federici and J.P. Zbilut (Conte *et al.,* 2004; 2004; Zbilut *et al.,* 1996; Zimatore *et al.,* 2000; Vena *et al.,* 2004), I was author of a number of papers showing violation of Lispchitz restriction in the sphere of biological dynamics. In a case we showed as the biological control mechanisms, formulated by a bicompartment model, violate Lipschitz conditions. The same violation was observed by us during human respiration by inspection of tracheal and pulmonary sounds. We gave a model of biological neuron implemented on the basis of a formulation violating Lipschitz condition. Heartbeat dynamics was analyzed by violation of Lipschitz condition. Previously, other authors gave very important contributions in this direction, and in particular A. Giuliani. For brevity in (Conte *et al.,* 2004; Zbilut *et al.,* 1996; Zimatore *et al.,* 2000; Vena *et al.,* 2004) we add only some of such fundamental contributing papers but really there is a lot of such important papers that our references does not take into account as it should be necessary. In short, here is a lot of ascertained violations while we are accustomed to continue to assume determinism as basic universal paradigma at the foundation of our reality and of our reasoning without exceptions.

In conclusion, we must be care in accepting determinism as universal paradigma in our reasoning and thinking. Note that the implications of such possible failure are of enormous importance for the argument that we have here in consideration. Take a differential equation as it was formulated by Zak starting with 1998. Write it as it follows

$$\frac{dx}{dt} + \alpha x^k = 0 \qquad (6)$$

with $k < 1$

Here the Lipschitz condition fails at the equilibrium point $x = 0$.

With $\alpha = -3/2$

The solutions are

$$x(t) = 0 \qquad (7)$$

and

$$x(t) \cong \pm (t - A)^{3/2} \qquad (8)$$

where A is an arbitrary parameter. The consequences of losing uniqueness of the solution are enormous for the problem that we have in consideration here. Consider a particle located at some summit at rest with the trivial solution holding for all times

$r(t) = 0$.

This mass simply remains at rest for all times (solution (7)). However, it exists also the other class of solutions given in the (8) so that we may write for any possible radial direction of the mass that

$r(t) \cong (t - A)^{3/2}$ for all times $t > A$

and

$r(t) = 0$ for all times $t \leq A$ **(9)**

Note the very important thing. We are describing the condition of a particle that is sitting at rest at the summit whereupon at an arbitrary time A it spontaneously moves in a certain arbitrary chosen radial direction. Note the language we are using: arbitrary time and spontaneously moves. Arbitrary time and spontaneous movement are two expressions that stimulate all our consideration in relation to the matter that we have here in consideration.

Arbitrary time spontaneous movement

We are adopting here terms as spontaneous movement after an arbitrary time. Are we using extreme mathematical model idealizations or are we remaining instead in the scheme of classical Newtonian physics? The solutions (9) are in perfect accord with Newton's first and second laws. In the absence of a net external force, a body is un-accelerated. In fact we obtain that for all times $t<A$, there is no net force applied, since the particle is at rest at position r = 0, it is un-accelerated. For all times $t>A$, there is a non-zero net force applied, since the mass is at positions r > 0 and the mass accelerates in accord with the second Newton law $F = ma$. Finally, when $t=A$, the direct computation of the mass acceleration from the equation (9) gives us

$$a(t) \cong \frac{3}{4}(t-A)^{-1/2} \qquad \textbf{(10)}$$

so that at $t=A$, the mass is still at the force-free summit, r=0, and the mass acceleration $a(0)$ is equal to zero. Again we have no force and no acceleration, as exactly the first Newton law requires. Acceleration exists only for times $t>A$. At time $t=A$ still acceleration does not exist. So in short we are in perfect accord with Newton's laws. We are not in the case of extreme idealization but on the other hand we are in front of two basic concepts of our reasoning: *we have here a model predicting an arbitrary time of spontaneous movement and the possibility of an arising spontaneous movement*. I interpret such last expression as essentially indicating that something happens spontaneously or, that is to say, without a cause. Something may happen in our reality spontaneously, that is to say, without cause. It is the physical model that is able to predict that the thing may go also in this manner.

Let me clear here. My aim is not to discuss here the mechanical features of the example that we have in consideration. This argument could be clearly debated on the pure physical level but this is not our purpose here. I intend to discuss about the principia regarding our reality. We have examples in which determinism is violated, causation seems to be violated, and still an example in which an arbitrary time and spontaneous arising movement are involved. They do not represent extreme idealizations as we have seen in detail, and so the arising conceptual view is perfectly accepted. We have also the sphere of spontaneous emergence in our reality, and we must take care of such conclusion when discussing the initial Eccles proposal about abstract field of probability causing neural events.

We may still comment about them clearing in detail our view point. The variable $X=(t-A)>0$ results to be essentially a random variable in our scheme. Thus we have a probability $P(X)$. *We may assert here that we have a variable expressing a fundamentally random length of time after giving origin to a spontaneous movement.*

Let us introduce a scheme of Clifford algebra as we use it usually in our Clifford scheme of quantum mechanics. Let us indicate the random X variable by the e_3 Clifford basic element so that it is a dichotomic variable being $e_3 \to +1$ for $t-A \le 0$ and $e_3 \to -1$ for $t-A>0$. It has a mean value $<e_3>$ that is regulated from its probability field:

$$<e_3> = (+1)p(+1) + (-1)p(-1) \qquad \textbf{(11)}$$

being $p(+1)$ the probability for e_3 to assume the value +1 and $p(-1)$ the probability for e_3 to assume the value –1.

So in conclusion. Have we acausation in generation of such spontaneous movement? I think otherwise. I retain that we have a probability field, as evidenced in (11), which is responsible of such spontaneous movement. In this manner we re-obtain the language of quantum mechanics that we introduced previously when we outlined that *the probability fields of quantum mechanics are causatively responsible for physical events. It is the abstract field of probabilities that in quantum mechanics determines events. In this manner it becomes evident that we may re-hook Margenau and Eccles position when these authors affirm that mental events trigger neural events. Mental events causing neural events analogously to the manner in which probability fields of quantum mechanics are causatively* *responsible for physical events,* as Wolf outlined**.** Margenau and Eccles conclusion do not pertain to extreme idealization according to the examples that we have given previously. I think that we should consider seriously their position when considering the spontaneous arising and link of mental events with neural events.

There is still another important feature that we have to outline here. *The model that we have introduced from the (6) to the (11) evidences the possible existing events marked by a fundamentally random length of time giving origin to spontaneous movement. It has thus profound implications under the psychological profile. We have here a clear indication on the manner in which it may arises the subjective experiences of space and of time in humans, and we know that the investigation of subjective experiences of space and time is at the core of consciousness research. For the first time we have here a mathematical formalism and a conceptual framework explaining it.*

Evidence n.2

Let us examine now the second theme that we introduced. The argument of the Aristotelian logic and its further developments used in our reasoning.

We have still some comment here. We have eminent mathematicians as Mumford and Jaynes who have dedicated a lot of their fundamental work to explain the very foundations of the mathematics and they have also given important contributions in the field of understanding the origins of our human functions of knowing and cognition. Munford has been very clear when explaining that the very foundations of mathematics should be reformulated on a stochastic basis (Mauford *et al.,* 2000). Mumford quotes the important contributions of Jaynes in this direction.

I totally agree with these authors when reaching the conclusion that Aristotelian logic must be seen as part of probability theory. I may understand that this conclusion is shocking but it is so.

Still, the important results do not stop here. There is a further step on. All the rules of probability theory can be deduced from a single primordial idea that involves in a natural way the fact that the information is involved. So we have two unavoidable starting points. The first is that Aristotelian logic is part of probability theory. The second is that probability theory can be deduced from the single primordial idea that involves the fact that the information is involved. This is obtained not on the basis of assumptions or elaborations, but instead on the basis of a precise theorem that is shown in mathematics and it is called the “Cox Theorem” (Cox, 1961). This theorem delineates one of the most important statements of this paper. Summarizing, it may be expressed in the following manner: Cox’s theorem states that, under certain assumptions, any measure of belief is isomorphic to a probability measure. Note the presence of terms as "belief" and "probability measure" in the proposition of this theorem. The conclusion that arises is clear and evident: We think in a probabilistic manner and all the previous reservations that we introduced about determinism, Aristotelian logic and its further developments receive now their complete and significant arrangement.

By the term plausibility we must intend a thing that seems or is apparently valid, likely, or acceptable or credible. In our reasoning we have to start from Cox basic postulates. Cox wanted his system to satisfy the following conditions:

1. Divisibility and comparability - The **plausibility** of a statement is a real number and is dependent on information we have related to the statement.
2. Common sense **- Plausibilities** should vary sensibly with the assessment of plausibilities in the model.
3. Consistency - If **the plausibility** of a statement can be derived in many ways, all the results must be equal.

We may give more convincing expressions of such statements:

We may say that Cox’s probability theory is not defined by precise axioms, but by three "desiderata":

(I) representations of plausibility are to be given by real numbers;

(II) plausibilities are to be in qualitative agreement with common sense;

(III) the plausibilities are to be "consistent", in the sense that anyone with the same information would assign the same real numbers to the plausibilities.

"Common sense" includes consistency with Aristotelian logic when statements are completely plausible or implausible. We may say that according to Cox, probability theory derives from such basic statements and it is a precise mathematical formulation of plausible reasoning.

The conclusion seems to be evident. We think in a probabilistic manner. To explain in detail: we have a given background knowledge. We call it the information I. Our brain works in a manner so that it has the ability to attribute a degree of plausibility to any new statement that is posed to the subject. Call it A. It may be a proposition, a statement or an hypothesis or other. Brain attributes a degree of plausibility to A on the basis of the background knowledge, I. I relates the human mental condition that he has in the moment in which A is posed. It relates his knowledge, feeling, rationality, emotions, and so on. This is to say that brains attributes a degree of plausibility by evaluating the real quantity $P(A/I)$ in the context dependent condition in which A is posed. In other terms, it evaluates the function $P(A/I)$. It is the probability (degree of plausibility) that brains attributes to A in subordination to the background information I that it has, including in I, as previously said, the contextual mental background in which A is posed.

In this manner the circle is closed. At the level of structure, according to Eccles, Beck and Walker, we have probability signed by the irreducible indeterminism of quantum mechanics. This is to say, we have quantum probability in quantum electron tunneling, in order to realize conjunction synapse. At the level of thinking, we have again probability that is the degree of plausibility that brains attributes to a given statement A. From one hand we have a probability field at the level of brain structure. As counterpart, we have probability as expression of the subjective plausibility. As example, that A is true given that I is true.

2. We Think In a Quantum Probabilistic Manner

The arguments that we have developed in the previous section are indicative. They are based only on the rigid elaboration of objective formal derivations and in any manner they take into account also only some of the real features of thinking conceived in a pure rational suit. Reasoning instead is based at all on human, individual, subjective, qualitative features. The observations of the previous sections may be useful to arrange in some manner the general problem of thinking but they are actually distant to approach actually the real features of our reasoning. Any psychologist will agree that humans are not expression of pure rationality. Humans don't always make the most rational decisions. There is a lot of studies that probe in a clear manner that even when logic and reasoning point in one direction, our thinking often chose the opposite direction. Our reasoning is motivated not only from rationality but from our personal bias. We may accept the basic framework to consider our reasoning as developing on probabilistic manner but the classical probabilistic procedure outlined in the previous section could result profoundly incomplete. It does not take in consideration all the components that actually assemble to determine our reasoning. We have also the wishful thinking, as example, in which we arrive to admit that some given statement is true only because we wish that it is true. Therefore, the approach of the previous section could be incomplete, not decisive. Just to support our thesis, using it only as an example, we may remember here that, according to Jung, humans have four psychological functions that are respectively the Thinking, the Feeling, the Sensing and the Intuition with the two basic attitudes that are the *Introversion* and the *Extraversion*. Humans use all such four psychological functions and attitudes in the structure of their Self and also for their cognitive performance. Each individual has his direct preferences for what functions and attitudes he uses predominantly. Thinking means first of all evaluating information or ideas rationally, logically. Certainly information develops a central role in our reasoning but also Feeling, just like Thinking, are matter for evaluating information. Sensing evaluates information also if takes information by the senses. Intuiting is a kind of perception, and based on information, works outside of the usual conscious processes. It works like sensing but comes from the complex integration of large amounts of information. So we have all such psychological functions and, as the reader may verify, each of such functions is marked by the term information. Our reasoning depends from all such psychological functions just to account only for the contributions indicated by Jung.

Arguments, in which it appears so evident the high complex nature of our reasoning, induce some psychologists often to consider that the physics is out from the possibility to describe such mechanisms. Other psychologists remain instead more possibility and in some manner claim that physics, and in particular quantum mechanics, could contribute in explaining the nature of our reasoning. My position is net. I am convinced that quantum mechanics is the first "physical theory" of reasoning and of our cognitive processes. I retain that quantum mechanics is the link between cognition and reasoning from one hand and the physical reality from the other hand. I do not forget here the split that occurred between psychology and physical sciences after the establishment of psychology as an independent discipline, and I am convinced that it contributed to the delay in acknowledging a possible link between such two disciplines. Still, I do not forget that N. Bohr, in formulating his basic principle of complementarity as foundation of quantum mechanics, realized such principle reading James as well as I do not ignore the consistent work that arose from the collaboration of Pauli with Jung. Certainly, I agree that these are only general arguments that may give little contribution to the more articulated problem to establish that quantum mechanics is the first theory of cognition and reasoning, but certainly, on the other hand, the profound meaning of contributions of founding fathers as Pauli, Bohr, James, and Jung and their high weight at the level of their intellectual profile, certainly cannot be ignored at all. My position is that quantum mechanics has origin in the logics; it is the first "physical theory" of cognition and reasoning. To support such thesis we should evidence that the concepts in our mind may combine following also quantum mechanics. Let us start introducing an experimental procedure.

Let us elaborate by considering the following experimental situation. We have an abstract or material entity that we call S that is constituted by a pair of separated sub entities S_1 and S_2 on which we may perform four experiments that we call respectively a_1, a_2, a_3, and a_4. Let us still consider that each of the experiments a_i $(i=1,2,3,4)$ has two possible outcomes, or $+1\,(r_+)$ or $-1\,(r_-)$. Still, continue to admit that some of these experiments may be performed together, respectively on S_1 and S_2, and we will call them coincidence experiments a_{ij} $(i,j=1,2,3,4)$. The experiment a_{ij} has four possible results, which are

$$a_i(r_+)a_j(r_+),\ a_i(r_+)a_j(r_-),\ a_i(r_-)a_j(r_+),\ a_i(r_-)a_j(r_-) \quad \textbf{(12)}$$

We may also introduce the expectation values for such coincidence experiments. We call them E_{ij}, and according to the definition, we have that

$$E_{ij}=(+1)p(\ a_i(r_+)a_j(r_+)\ )+(-1)p(\ a_i(r_+)a_j(r_-))+(-1)p(\ a_i(r_-)a_j(r_+))+(+1)p(a_i(r_-)a_j(r_-)) \quad \textbf{(13)}$$

Obviously, p_{ij} means the probability that the coincidence experiment a_{ij} gives the outcomes $r_i r_j$ while, generally speaking, p_i will represent the probability that the single experiment a_i will give outcome r_i $(i,j=+,-)$

This is a basic scheme that in several our previous papers we have discussed in the framework of the so called Clifford algebra by which we have realized a rough or "bare bone skeleton" of quantum mechanics (Conte, 2000; 2010; Conte *et al.,* 2006). We will not discuss further such elaboration here addressing the reader to the above quoted papers for a close examination.

In the forthcoming steps of this paper we will describe the physical conditions in which by using the (12) and the (13), we may derive the celebrated Bell inequality which states explicitly

$$\left|E_{13}-E_{14}\right|+\left|E_{23}+E_{24}\right|\leq 2 \qquad \textbf{(14)}$$

Summarizing, we have an entity S constituted by two separated components entities S_1 and S_2. We may perform an experiment a_1 on S_1 obtaining as result r_+ or r_-. We may still perform an experiment a_2 on S_1 still obtaining as result or r_+ or r_-. We may perform an experiment a_3 on S_2 and it may be also similar to a_1 on S_1 with possible results r_+ or r_-, and finally an experiment a_4 on S_2 that may be similar to a_2 on S_1 with possible results r_+ or r_-. Now, the experiment a_1 may be performed in coincidence with the experiments a_3 and a_4, and thus we denote such coincidence experiments by a_{13} and a_{14} respectively, and thus obtaining E_{13} and E_{14}. We may also perform the coincidence experiments a_{23} and a_{24} obtaining E_{23} and E_{24}. All such expectation values are considered in the previous (14).

In quantum mechanics, we choose the set of observable properties of a quantum entity to which we are interested. These constitute the state of the entity. We also define a state space, which delineates the possible states of the entity. A quantum entity is described using not just a state space but also a set of measurement contexts. The algebraic structure of the state space is given by the vector space structure of the complex Hilbert space: states are represented by unit vectors, and measurement contexts by self-adjoint operators.

The crucial notion on which we may fix our consideration is the notion of quantum entanglement. With reference to entity S and to the two composing subentities S_1 and S_2, one says that a quantum entity is *entangled* if it is a composite of subentities that no more can be factorized in their components that of course may can be identified only by a separating measurement. When a measurement is performed on the entangled entity, its state changes probabilistically, and this change of state is called *quantum collapse*.

In pure quantum mechanics, if H_1 is the Hilbert space representing the state space of the first subentity, and H_2 the Hilbert space representing the state space of the second subentity, the state *i.e.*, $H_1 \otimes H_2$. These are standard notions in quantum mechanics. Aerts has repeatedly outlined an important feature at cognitive level (Aerts *et al.,* 2011; 2005a; 2005b; 2000; Gabora *et al.,* 2009; 2007; 2002; Bruza *et al.,* 2009). He discusses in detail that the tensor product always generates new states with new properties, specifically the entangled states. Thus the space of the composite system is not the Cartesian product, as in classical physics, but the tensor product, and it is used to describe the spontaneous generation of new states with new properties.

Entanglement was recognized early as one of the key features of quantum mechanics. Entanglement still can be described as the correlation between distinct subsystems and such correlation cannot be created by local actions on each subsystem separately. The advantage given by quantum entanglement relies on the crucial premise that it cannot be reproduced by any classical theory (Aerts *et al.,* 2011; 2005a; 2005b; 2000; Gabora *et al.,* 2009; 2007; 2002; Bruza *et al.,* 2009). Despite the fact that the possibility of quantum entanglement was acknowledged almost as soon as quantum theory was discovered, it is only in recent years that consideration has been given to finding methods to quantify it, and to analyze it in the framework of the cognitive level (Aerts *et al.,* 2011; 2005a; 2005b; 2000; Gabora *et al.,* 2009; 2007; 2002; Bruza *et al.,* 2009). Historically, the Bell inequalities are seen as a means of determining whether a two quantum state system is entangled.

It was known that the larger the violation of the Bell inequality is, the more the entanglement is present in the system. This led to the perception that to some degree the Bell inequalities were a measure of entanglement in such systems.

In this manner we arrive to the point that we can use the violation of Bell inequality as an experimental indication for the presence of a quantum structure. If Bell inequalities are satisfied for a set of probabilities connected to outcomes of the previously considered experiments, there exists a classical Kolmogorovian probability model. In such model the probability can be explained as due to a lack of knowledge about the precise state of the system under consideration. If, on the other hand, Bell inequalities are violated, as shown in (Aerts *et al.,* 2011; 2005a; 2005b; 2000; Gabora *et al.,* 2009; 2007; 2002; Bruza *et al.,* 2009; Pitowsky, 1989), no such classical Kolmogorovian probability model exists. Quantum states arise as having ontological potentiality and thus intrinsic irreducible indeterminism. Probabilities in this case $i = 1,2,3$) are involved as non classical and thus become the non classical probabilities, that is to say, the quantum probabilities that characterize the sphere of quantum ontological processes. This reason because to examine the (14) is so important.

D. Aerts was the first to consider the opportunity to analyze concepts and their combination through quantum mechanics showing their possibility to violate Bell inequality (Aerts *et al.,* 2011; 2005a; 2005b; 2000; Gabora *et al.,* 2009; 2007; 2002; Bruza *et al.,* 2009). We will follow this scheme but based once again on our basic scheme with Clifford algebra. Let us consider two Clifford sets based on the following basic elements:

(E_{01}, E_{02}, E_{03}),

$E_{0i}^2 = 1$ ($i = 1,2,3$), $E_{0i}E_{0j} = -E_{0j}E_{0i}$;

$E_{0i}E_{0j} = iE_{0k}$ ($i \neq j \neq k$)

and cyclic permutation of (1, 2, 3);

(E_{10}, E_{20}, E_{30}),

$E_{i0}^2 = 1$ ($i = 1,2,3$), $E_{i0}E_{j0} = -E_{j0}E_{i0}$;

$E_{i0}E_{j0} = iE_{k0}$ ($i \neq j \neq k$)

and cyclic permutation of (1, 2, 3); **(15)**

E_{0i} and E_{i0} are basic abstract entities of our mind representing concepts. Each basic element, according to the (15), may assume the numerical values or $+1$ or -1. Let us admit we ask to a subject to concentrate himself on the class of medical specialization. Call A the specializations : dentist and cardiologist and both such specializations are identified by E_{03}. This is to say that he considers $E_{03} \to +1$ when he selects dentist while instead he considers $E_{03} \to -1$ when he selects cardiologist. Now introduce the second group of specializations, A': anaesthetist and urologist. Both such specializations are identified by E_{02}, and he considers $E_{02} \to +1$ when he selects anaesthetist while instead he considers $E_{02} \to -1$ when he selects urologist. Now we consider concepts connected to anatomic structures. Call B: heart and teeth. Both they are identified by E_{30}. The subject considers $E_{30} \to +1$ when he selects heart while instead he selects $E_{30} \to -1$ when he selects teeth.

Finally we call B': bones and prostate. Both they are identified by E_{20}. The subject considers $E_{20} \to +1$ when he selects bones while instead he considers $E_{20} \to -1$ when he selects prostate. In this manner we have four conceptual groups, A, A', B, B', and we may analyze how it is the conceptual behavior of the subject when he combines such conceptual groups under the common conceptual requirement of "to cure a disease". The subject may perform such combinations: $AB, AB', A'B, A'B'$. In algebraic terms we have $AB = E_{03}E_{30}$, $AB' = E_{03}E_{20}$, $A'B = E_{02}E_{30}$, $A'B' = E_{02}E_{20}$. According to our rules of our quantum Clifford algebraic scheme, we may also calculate the expectation values, writing $< E_{03}E_{30} >$,

$< E_{03}E_{20} >$,

$< E_{02}E_{30} >$,

$< E_{02}E_{20} >$,

and assuming conceptual independence, it results that each of such expectation values may assume or the $+1$ or the value -1. On the other hand the Bell inequality says that

$$\left| < E_{02}E_{20} > + < E_{02}E_{30} > \right| + \left| < E_{03}E_{20} > - < E_{03}E_{30} > \right| \leq 2 \qquad \textbf{(16)}$$

Of course the (16) may be re-written in the following manner:

$$\left|\frac{<(E_{02}+E_{20})^2>-2}{2}+\frac{<(E_{02}+E_{30})^2>-2}{2}\right| + \left|\frac{<(E_{03}+E_{20})^2>-2}{2}-\frac{<(E_{03}+E_{30})^2>-2}{2}\right| \leq 2 \quad (17)$$

Note that we may have

$E_{03}\to+1$, $E_{30}\to-1$, $E_{20}\to+1$

$E_{03}\to-1$, $E_{30}\to+1$;

$E_{02}\to-1$, $E_{20}\to-1$; **(18)**

$E_{02}\to+1$, $E_{30}\to+1$.

Let us observe that with such values inserted in the (17), we obtain the final value of 4 and this is to say that Bell inequality is violated. We have quantum entanglement. Let us examine under the conceptual profile what actually happens. First let us translate the case $<AB>=+1$. Return to our previously mentioned notion of plausibility. We ask to the subject what conceptual combinations he finds plausible:

The dentist cures the heart

The cardiologist cures the teeth

Otherwise let us translate the case $<AB>=-1$. We ask to the subject what conceptual combinations he finds plausible.

The dentist cures the teeth

The cardiologist cures the heart.

Without doubts, inspecting both such pair of sentences the subject should find more plausible the case that we write

$<E_{03}E_{30}> = <AB>=-1$. **(19)**

Let us now examine the case $<AB'>=+1$. It is

The dentist cures the bones

The cardiologist cures the prostate

In the case $<AB'>=-1$ it is

The cardiologist cures the bones

The Dentist cures the prostate.

In this case he should find more plausible the case $<E_{03}E_{20}> = <AB'>=+1$.

Let us examine now the case $<A'B>=+1$. It is

The anaesthetist cures the heart

The urologist cures the teeth.

In the case $<A'B>=-1$, it is

The anaesthetist cures the teeth

The urologist cures the heart

Also in this case the subject should find more plausible the case

$<E_{02}E_{30}> = <A'B>=+1$. **(20)**

Finally, let us consider the case $<A'B'>=+1$. It is

The anaesthetist cures the bones

The urologist cures the prostate

Instead, in the case $<A'B'>=-1$, it is

The anaesthetist cures the prostate

The urologist cures the bones.

Again in this final case the subject should find more plausible the case $<E_{02}E_{20}> = <A'B'>=+1$.

Inserting such values in the (16), one finds that in perfect accord with the (18) the Bell inequality is violated, gives value equal to +4, and thus we may conclude that in such conceptual case the subject combines concepts and does plausibility, using quantum entanglement. In conclusion we have reached the evidence n.3

Evidence n.3

Combinations of concepts may follow quantum entanglement

We re-outline here. Such studies are due to D. Aerts. We have here reformulated the question using a proper example using the Clifford algebra that usually we engage as rough bare bone skeleton of quantum mechanics.

We have to consider now the problem of the Self. May we introduce a mathematical-physical model of the Self?

Also if we mentioned previously that the first psychological studies and physics went both in psychology at the first starting of this discipline, today they are seen together so infrequently. May be that when physics is considered so linked to mathematics as it is the case of the present elaboration, both fields seem so abstract that describing one in terms of the other is seen soon with some prejudice and considered notable of giving some advantage.

Previously we have outlined instead that these two ways, particularly when physics is supported from strong mathematics, represent two ways of thinking developed integrally in the same individual. However, Freud, as example, developed his results using symbols, analogies, figures in the world of the arts and of the literature but never he used mathematics or physics. Instead we have quoted previously eminent figures of mathematicians that have given fundamental contributions having had so much to say about the workings of mind and Descartes gave in my modest opinion the first psychological legacy to physical knowledge by his Cogito ergo Sum that in some manner will represent the anticipated conclusion to which I will arrive examining some recent results obtained by me in quantum mechanics.

In this paper I would be able to indicate some result in the direction of mapping the structure of the self by using quantum mechanics: To present some modeling example aiming to match the human experience of selfhood. I am encouraged by this way since previously I gave examples of spontaneous arising abstract probability fields in accord with the genuine nature of mental events as they were postulated by Margenau and Eccles.

In modeling the Self I outline here his first nature that is reflectivity. Self is by its nature self-referential. It is at once subject and object, observer and observed of itself as well as of the others. This attitude has often lead psychologists to consider dualistic theories. Self-observation is the key concept here. Lefebvre's mathematical approach to social psychology is often referred to as reflexive theory - presumably due to the reflexive nature of taking into account subjects' self-image(s). I would obtain here that the centuries-old philosophical and psychological idea that man has an image of the self containing an image of the self obtains a new advance in the mathematical-physical model of the subject possessing reflection that I outline here. One assumption underlying the model is that the subject tends to generate patterns of behavior such that some kind of similarity is established between the subject himself and his second order image of the self.

Still, quantum mechanics is based on its basic formulation of intrinsic and irreducible indeterminism.

Would psychologists speak about indetermination or inter-determination? Many disorders of the Self consist in the spreading between the subjective and objective features of the self. In hallucinations, as example, dreams, imaginations the subjective and objective features separate. In the intrinsic undependability of self-observation, a dose of intrinsic and irreducible indetermination arises for us all and we have unconscious as relevant counterpart. At the extreme limits we have the whole spectrum of psychopathology. So, the importance of a model arises.

As previously said, we use Clifford algebra to represent a bare bone skeleton of quantum mechanics.

Let us give an example of our approach. Let us introduce three basic algebraic abstract elements e_i , $i = 1,2,3$, having the following basic features:

1) $e_i^2 = 1$ and 2) $e_i e_j = -e_j e_i = i e_k$ with $i, j, k = 1,2,3$, $ijk =$ permutation of 1, 2, 3 and

$$i^2 = -1 . \qquad \textbf{(21)}$$

We see that the axioms 1) and 2) introduce the two basic requirements that we invoke for quantum mechanics: potentiality and non commutative. The first axiom in fact introduces an abstract entity, e_i, but at the same time fixes that its square is 1. This is to say that to each e_i with $i = 1,2,3$, under particular conditions in such an algebra, may correspond or the value +1 or the value -1. For each e_i we have the ontological potentiality to link one of such possible numerical values. The second axiom introduces non commutative for $e_i (i = 1,2,3)$.

The abstract elements e_i are marked by irreducible, intrinsic indetermination. We may calculate their mean values, $< e_i >$ considering the probabilities for +1 or for -1 values, and writing

$$< e_1 >= (+1)p(+1) + (-1)p(-1) ,$$

$< e_2 >= (+1)p(+1) + (-1)p(-1)$,

$< e_3 >= (+1)p(+1) + (-1)p(-1)$ (22)

where $p(+1)$ and $p(-1)$ represent the probabilities for +1 and -1 values, respectively, with $p(+1)+p(-1)=1$. The quantum like features of this algebra may be synthesized in the following equation that we discussed in our previous work (Conte, 2000; 2010; Conte *et al.,* 2006):

$< e_1 >^2 + < e_2 >^2 + < e_3 >^2 \le 1$ (**23**)

In this manner a quantum mechanical scheme may be represented by such algebra. We may introduce the well known Pauli matrices at order n=2 as representative for the basic elements e_i. This is an important operation since, from one hand, it helps us to identify some hidden features of our algebra, and, on the other hand, it introduces for the first time the possibility of a self-referential operation. Let us proceed with the aid of an example. Let us suppose that in the operation of a progressive description of some entity or structure, we have arrived at the condition that two dichotomous variables A and B are actually required in order to characterize it. We may use the matrix representation of the basic elements e_i and we may realize some new algebraic elements given by the direct product of matrices. In this case, we will have new basic elements in the following manner:

$E_{oi} = I \otimes e_i$ and $E_{io} = e_i \otimes I$ being I the unit matrix, $i = 1,2,3.$ (**24**)

Note that E_{0i} and E_{i0} will satisfy the same rules that were given in 1) and 2) for e_i. In detail we will have that

$E_{0i}^2 = 1$, $E_{0i}E_{0j} = iE_{0k}$, and $E_{i0}^2 = 1$, and $E_{io}E_{jo} = iE_{k0}$. (**25**)

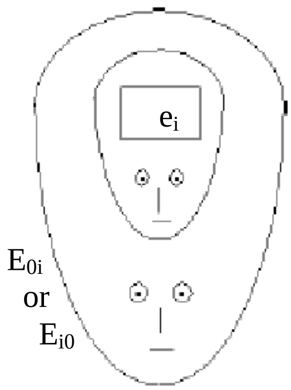
It is important to observe that we will have also that $E_{0i}E_{j0} = E_{j0}E_{i0}$ for any (i, j) and $i = 1,2,3; j = 1,2,3$.

As required, we have now two dichotomous variables, E_{0i} and E_{i0}, $i = 1,2,3,$ to describe. Let us consider still that e_i are the basic elements of our algebra given at order n=2 in our isomorphism while E_{0i} and E_{i0} are the same basic elements but at order n=4.

In the case of the Self, we are accustomed to conceive the simplest features of observer and observed that in our interpretation become the inside and outside respectively. The fact that they are separate and at the same time have unity, appears impossible to us but actually it is due to an artifact of our traditional point of view on this matter. This is precisely the question with all dualism in psychology. However this is a matter that may be overcome accepting a less ingenuous and less modest vision of our reality. Think as example about the concept of quantum entanglement in quantum mechanics or consider $E_{0i}E_{j0} = E_{j0}E_{i0}$ of our algebraic basic scheme. They give rise to the new algebraic basic set E_{ji} or E_{ij}.

Note that for the first time we have also introduced a self referential mathematical formalism. To explain such a referential mathematical operation, let us return to our basic algebraic scheme but evidencing what Lefebvre (Lefebvre, 2002) recently outlined. As we know, the central topic of Western philosophy, starting with John Locke, was the problem of representing mentally one's own thoughts and feelings. Actually, it is a very difficult concept to represent. This is the reason to use here a pictorial representation, the same figure that Lefebvre introduced to describe his formulation (Lefebvre, 2002). We may express self attitude through the reflection. A subject having reflection may be conceived as a miniature human figure with the image of the self inside his head. We recover it here in the following figure. It represents with care the subject with reflection. We prefer to call it the picture of a subject having perception of itself. In this figure, following VA. Lefebvre, we may say that inside the subject's inner domain, there is an image of the self with its own inner domain. An image of the self is traditionally regarded as the result of the subject's conscious constructive activity.

Let us analyze now the mathematical operation given in (24). It is the faithful correspondent of the self-picture given in figure in which, in fact, E_{0i}, for example, or also E_{i0}, contain in their inside that image of itself that is e_i. We may conclude that, at least for our present possibilities of understanding what the self is and its self-perception represents, we have for the first time identified a basic algebraic scheme and the corresponding mathematical operations to represent it.



This completes our brief exposition on self introduced by a bare bone skeleton of quantum mechanics using the Clifford algebra. Note the important interface that we are delineating. As repeatedly outlined we use a bare bone skeleton of quantum mechanics using the Clifford algebra. The basic elements are the e_i. Note. They do not represent traditional quantum observables but abstract entities. Of course it is traditionally accepted in standard quantum mechanics to connect to the operators e_i, the spin components. We know that previously other authors (Hu *et al.,* 2002; 2004) outlined the role of spin as self-referential variable and its possible role on the advent of consciousness. They introduced the spin-mediated consciousness theory. We will discuss in detail such feature in the last section entitled "Further Advances", but we may anticipate here that matter and its physical properties must be considered to be interfaced with cognitive feature. This could be one of the profound reasons because in their papers in (Hu *et al.,* 2002; 2004) it was evidenced the so important role for the spin. In particular these authors arrived to give explanation of its role and function, giving also some important neurophysiological correlates.

Now we may pass to consider a possible theory of personality. In Jungian theory, the Self is one of the archetypes. The coherent whole unifies consciousness and unconscious of a person. The Self, according to Jung, is realized as the product of individuation, which in Jungian view is the process of integrating one's personality.

What distinguishes Jungian psychology is the idea that there are two centers of the personality. The ego is the center of consciousness, whereas the Self is the center of the total personality, which includes consciousness, the unconscious, and the ego. The Self is both the whole and the center. While the ego is a self-contained little circle off the center contained within the whole, the Self accounts for the observed individual differences in people personality. A large number of personality theorists have contributed to the field but there is no integrated theory and we are left with a variety of individual approaches.

Let us return briefly to the question, before mentioned, of basic four psychological functions of Thinking, Feeling, Sensing, Intuiting and Attitudes (Introversion and Extraversion) as they were considered by Jung. Certainly, if I claim here that such psychological function are linked and inter-related with attitudes in humans, I do a so general and unspecific statement that all the psychologists will agree with me. However, an interesting thing could be to advance such so phenomenological approach only, and attempt to give to the basic four psychological functions and to the attitudes a theoretical formulation so that we may experiment about, and obtain precise and quantitative results.

As example, a question that I pose to myself is the following: once again could psychological functions be quantum entangled with attitudes? If such kind of possible correlations should be evidenced, I certainly will obtain first of all a further evidence of the effective role explained from quantum mechanics in brain and mind, and, in addition, a new quantum model of Jung theory of personality should arise, this time based on the principles of a well defined physical theory.

Let us indicate me the Feeling by F, the thinking by T, the sensing by S and the Intuition by I. Still I call E the extroversion and I_1 the introversion.

My approach should be well known to the reader by this time. I introduce now some Clifford basic elements. I call the Thinking function (T) by E_{03}. It is a dichotomous that as previously explained, may admit values or +1

or -1. $E_{03} \to +1$ means that the subject is Thinking. $E_{03} \to -1$ means that he is Feeling (F).

So I have that

$$T = -F = E_{03} \qquad (26)$$

This is the quantum scheme for rational functions. Now I introduce the irrational functions. I call the Sensing (S) by the Clifford basic element E_{01} to which again are linked the values ± 1. $E_{01} \to +1$ means that the subject is Sensing while instead $E_{01} \to -1$ means that he is Intuitive (I). So I have

$$S = -I = E_{01} \ . \qquad (27)$$

These are the four psychological functions. Let us now introduce the attitudes of the Self, calling E extroversion and I_1 introversion. Let us consider another algebraic Clifford Element

$$E = -I_1 = E_{30} \qquad (28)$$

$E_{30} \to +1$ means extroversion, otherwise $E_{30} \to -1$ means introversion.

Finally, let us consider another Clifford basic element representing that accounts for states of explicit intermediation between extraversion E and Introversion I_1. I call it M, and I pose

$$M = E_{10} \qquad (29)$$

with the realization that it assumes $E_{10} \to +1$ when the subject is in a state of equal superposition of pure extroverted and pure introverted condition while instead we have $E_{10} \to -1$ otherwise.

In this manner we have realized two basic features. The first is that by introducing the (26), we have fixed that the rational functions are opposites from each other and, considering the (27), we have admitted that also the irrational functions are opposites from each other.

Obviously, consider that, using the (26) and the (27), we enter by Clifford algebra in a quantum bare bone skeleton of quantum mechanics. This is to say that rational as well as irrational functions have an irreducible intrinsic indetermination in their state. This is to say that the person has an ontological potentiality, a quantum superposition of alternatives, to be T or F becoming actually T or F when his Self is submitted to direct self or outside direct observation. The same thing happens for psychological functions S and I being the person in a superposition of such states and becoming actually S or I. Obviously, the selection of the state T or F, and, respectively, S or I is only a matter of probability that is enhanced in favoring one psychological function respect to the other in dependence of the inner structure of his Self and of the context in which the self is under direct observation.

This is the quantum scheme of the approach. In other terms both superior and inferior functions coexist, and it is only a matter of our inner developed structure and of the instantaneous context that, probabilistically speaking, one function results prevailing on the other in our subjective dynamics.

Fixed such important conceptual points, let us attempt to give soon some result confirming possibly that we are formulating a theory in a correct direction. Let us calculate the expectation value (mean value, of T, F, S, and I). Looking at our basic relation of Clifford algebraic scheme of quantum mechanics given in the (23), we obtain immediately that

$$<T> = \cos\vartheta \, , \ <F> = -\cos\vartheta \, ,$$
$$<S> = sen\vartheta \, , \ <I> = -sen\vartheta \qquad (30)$$

where ϑ is an arbitrary angle ranging from $-\pi$ to π

Let us schematize the results of the (30) in Fig.3. We obtain the behaviors of the expectation values for such psychological functions.

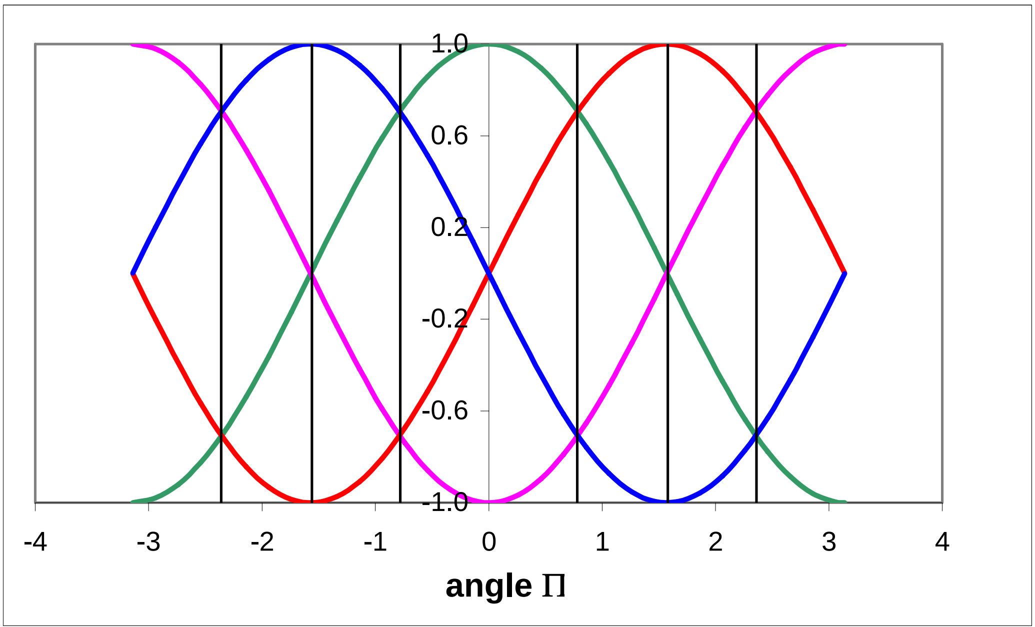


Figure 3. Expectation values of the four psychological functions.

It is easily observed that we obtain eight corresponding sections:

1) F>I>S>T
2) I>F>T>S
3) I>T>F>S
4) T>I>S>F
5) T>S>I>F
6) S>T>F>I
7) S>F>T>I
8) F>S>I>T

There is no doubt that our approach reproduces perfectly the eight different proportions that were identified also by Jung theory when he characterized the superior and secondary psychological functions of a subject. Remember that he outlined that we just have them in different proportions. We have a superior function which we prefer and it is best developed in us, and a secondary function of which we are aware and we use in support of our superior function. The personality of a person conflicts if the Self has to realize two opponent functions in the same attitude. Here it is one of the interesting features of such our results obtained by Clifford algebraic scheme of quantum mechanics.

By using the (30) we may now experimentally estimate the values of the possible ratios of T, F, S, I, and evaluate their balancing in normal as well in pathological conditions. This is an interesting step on.

This last conclusion completes our exposition on the Jung four psychological functions as elaborated by a bare bone skeleton of quantum mechanics.

Now the attitudes of the Self. The different attitudes of the Self may be extraversed or introversed, and they have been quantum mechanically expressed by us in the (29) and the (30). According to our quantum language, as previously for the four psychological functions, also here the situation is now conceptually changed. We may have pure extroversed or pure introversed states but we may also have the ontological true potentiality, signed from irreducible indeterminism, of potential superpositions of extroverted and introverted states. Here, this feature is also enhanced from the presence of the Clifford algebraic element

$$M = E_{10}$$

to which we attribute the numerical value of -1 if the subject always collapses to a possible state of extraversion or intraversion while it still remains to be +1 if the subject remains in an uncollapsed state of equal superposition of pure introvetrted and pure extroverted states. Also in this case we may calculated the mean values obtaining

$$< E >= \cos\phi \,, \quad < I >= -\cos\phi \,,$$
$$< M >= sen\phi \qquad (\mathbf{31})$$

Under the profile of the experimental investigation we may repeat here all that we have previously outlined for the psychological functions. We may explore the attitudes of the Self and his balancing. It is relevant to outline here further the importance of such acquired possibilities under the basic theoretical profile of the elaboration as well as in the case of analyzing possible implications under the clinical profile.

Now a step one. It may be useful to repeat here the notion of quantum entanglement that we have also prospected previously. Using very simple terms we may say that quantum entanglement is a pure quantum phenomenon in which the states of two or more objects or entities anyway separated, remain linked together so that one object can no longer be described without considering its counterpart. A quantum interconnection maintains between the two components also for any space distance separation between the two separated objects, leading to a net correlation between measurable observable properties of such two or more components. We need to re outline here that such very extraordinary property of correlation at distance relates only quantum entanglement that is exhibited only from systems subjected to the principles and to the rules of quantum mechanics. We need the previously mentioned Bell inequality. If it is violated, we have quantum entanglement.

Our attempt is to verify if or not Jung theory has a possible quantum formulation. By this way we may admit that human subjects in some conditions realize quantum entanglement in the sense that psychological functions are entangled with Self-attitudes and we may write Bell inequality linking psychological functions and attitudes. With clear evidence of the used symbolism, we write in this case the Bell inequality in this manner

$$\left|E(M,T) - E(M,S)\right| + \left|E(E,T) + E(E,S)\right| \le 2 \quad (\mathbf{32})$$

E states for expectation value. *M, T, S, E* state for attitudes M, and E and for psychological functions T and S. All we know about the MBTI that is to say the Myers-Briggs Type Indicator. We may use MBTI to classify the personality of the subject adopting some predefined sentences.

I give here a brief introduction to the experiment that is still in progress, and it will be exposed in detail elsewhere (Conte, 2010d).

My colleagues and I decided to use the MBTI to submit the (32) to experimental verification in order to evaluate if or not we may speak about quantum entanglement between psychological functions and attitudes in human subjects. We decided to perform an experiment that I prepared with A. Khrennikov, R. Blutner, A. Federici, O. Todarello, V. Laterza, A. Losurdo, and S. Goffredo. We thought the experiment in the following manner. Using the sentences given in the MBTI, we prepared possible pairs of sentences $(M,T),(M,S),(E,T),(E,S)$ coupling them in a computer archive. Male and female normal subjects were selected with age ranging from twenty to thirty years old. Each subject was subject to simultaneous sentences (M,T), soon after (M,S), then (E,T) and finally (E,S), each pair of sentences given to subject after a short time from the other. Each pair of sentences was selected at random by the computer from the previously arranged archive and given to the subject. In this manner we calculated $E(M,T), E(M,S), E(E,T), E(E,S)$ for each subject. For each person we repeated the experiment three times selecting at random every time the pairs of sentences. Each administration was given to the subject after a period of at rest for the subject of about 15 minutes. The experiment is still in progress, thus we are in the condition to give here only some preliminary indications, but of course we may give us some important anticipation. As previously said a group of three psychologists, specialized in the administration of psychological tests, were active in the experiment. One of them found that the Bell inequality was violated in the 58% of the investigated cases, the other psychologist found instead Bell violation in the 62% of cases, and the third psychologist found a violation in the 73% cases. As said, also if such data do not give us still a complete indication, however the experimental seems to evidence quite clearly the final results that we may expect. Subjects showed in percent a violation of Bell inequality and this is to say that in such case psychological functions and attitudes in these subjects gave quantum entanglement.

Evidence n 4

The Self and psychological functions in humans, as considered by Jung, seems to agree with the rules of quantum mechanics. In particular, psychological functions and attitudes seem to realize in a large percentage of cases quantum entanglement. I have not stated first such my specification to give here more emphasis and importance: I must now complete here this discussion evidencing that the first idea to use two qubits for Jung's theory of personality is due to Reinhard Blutner, and Elena Hochnadel. They started this very important work, based on this excellent idea in 2009 with a number of preprints available on line (www.illc.uva.nl) and recently they have also published this elaboration on Cognitive Systems Research. I ask to the reader to read these papers with great interest because, in my modest opinion, they are excellent and of basic importance (Blutner *et al.,* 2010).

I take now a further step one. Starting with 1972, I began to elaborate a quantum mechanical approach with relation to the field of the Clifford algebra. In particular in 1983 (Conte, 1983; Conte *et al.,* 2009), I advanced the basic elaboration having the finality to reconstruct the quantum wave function, if existing, for a system, starting from the experimental data.

Why such elaboration may be so important here! Obviously, physicists study usually quantum systems and rarely they are interested to reconstruct the quantum wave function for the considered system. Obviously they are interested in selecting some quantum observable of interest and thus in estimating the possible eigen values and the corresponding probabilities of such obtainable results. In our cognitive-psychological framework, the background is totally different.

A priori we do not know anything about the human cognitive system that we are exploring by our tests. We do not know if it is classical or if, instead, quantum mechanics has a determinant role. We need a criterium to select between such alternatives and it must be so robust that on its basis we must accept or not if mental states followed or not quantum mechanics during the performed experiment. I think that we have an unquestionable criterium to adopt. It runs as it follows: if we are able to reconstruct quantum wave function of such psychological system, we are in the condition to conclude in an unquestionable manner that such system is governed by quantum mechanics. There are in psychology several well known techniques, also statistically, to select if a given model at cognitive level is better than another. But in my opinion they are not sufficient in this case. If we arrange an experiment, and we have the finality to ascertain that this experiment is involving a human cognitive system that is supported from quantum mechanics, we have consequently to proof only one thing .... and precisely ... that such system admits a well defined and correctly calculated quantum wave function, reconstructed from the obtained experimental data. It is certainly true that in the domain of quantum mechanics, the wave function is that "dark object" that cannot be observed directly. It does not represent a quantum observable in the traditional sense of this term, but, if existing, we may still reconstruct it starting from measurements, thus from experimental data. Therefore, the only robust criteria is to reconstruct quantum wave function, and, if existing and reconstructed correctly, we have the decisive element to conclude about the fundamental role of quantum mechanics in the system having as counterpart the data obtained by the experiment. This was essentially the aim of my elaboration in 1983.

Starting with 1992 Andrei Yuri Khrennikov (2009) gave an appropriate quantum-like description and treatment of cognitive systems, also suitable for application by experiments at cognitive level. On the basis of such indications, with Khrennikov, Todarello, Federici, in years from 1993 to 2010, I have given a lot of experimental confirmations on the existing mental states following quantum mechanics at the perceptive-cognitive level in humans (Conte 2008; Conte *et al.,* 2003; 2008; 2007; 2009a; 2009b; 2008a; 2008b). Let us explain briefly the problem in a rough manner but satisfactory to be followed also for scholars that have not a deep knowledge of quantum mechanics.

Let us select two psychological tests A and B to be given to a subject. Let us realize such A and B tests so that such variables A, B are dichotomic. This is to say that they may assume only two values (± 1) being, as example, +1 Yes and –1 Not. Let us admit now that we select two appropriate populations of subjects, the group C and the group D. To each component of the group C, we give the test A. Each subject will answer with Yes or Not so that at the end of the experiment we will have the probability $p(A=+1)$ and the probability $p(A=-1)$ with $p(A=+1)+p(A=-1)=1$.

Now we consider the group D. To each of such subjects we give first the test B immediately followed by the test A.

In this case we will estimate the probabilities $p(A=+1/B=+1)$, $p(A=+1/B=-1)$, $p(A=-1/B=+1)$, and p(A=-1/B=-1)
with $p(A=+1/B=+1)+p(A=-1/B=+1)=1$, and $p(A=+1/B=-1)+p(A=-1/B=-1)=1$.

In order to exemplify the argument I will not enter here in the discussion on the basic foundations of probability theory, on Kolmogorov approach and on other very important basic foundations of probability calculus. Of course Andrei Khrennikov has deepened largely such basic argument also in his recent book that I have previously quoted. I will remain at the most simple basic step that, as it is well known, is represented by the Bayes theorem. As we know, according to Bayes, we obtain that

$$p(A=+1)=p(B=+1)p(A=+1/B=+1) + p(B=-1)p(A=+1/B=-1) \qquad \textbf{(33)}$$

A similar relation holds for $p(A=-1)$.

What is now the basic foundation of our experiments! It is that the (33) pertains to classical probability theory while instead it is violated in the case of quantum mechanics. In quantum mechanics a further quantum interference term appears and, instead of the (33), we obtain

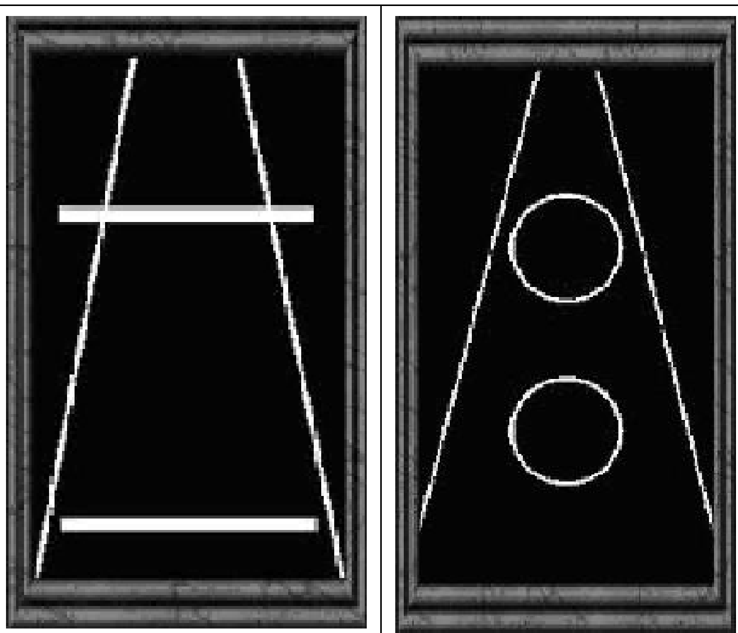
$$p(A=+1)=p(B=+1)p(A=+1/B=+1) + p(B=-1)p(A=+1/B=-1) + 2\sqrt{p(B=+1)p(A=+1/B=+1)p(B=-1)p(A=+1/B=-1)}\cos J \qquad \textbf{(34)}$$

Therefore, in the case of quantum interference, we have a further term:

$$2\sqrt{p(B=+1)p(A=+1/B=+1)p(B=-1)p(A=+1/B=-1)}\cos\vartheta$$

respect to the classical case given in (35). Obviously, a similar relation hold in the case of $p(A=-1)$.

Now, I will not discuss here the quantum interference for brevity. The reader is again sent to deep this argument. I will limit myself to outline that it is at the basic foundation of quantum mechanics. This theory runs about two basic foundations, one is the irreducible indeterminism and the other is the quantum interference. There is no way to escape to quantum interference if the investigated phenomenon drops into the domain of quantum mechanics. In conclusion we have reached a very power scheme to verify if mental states follow or not quantum mechanics. We may perform the previous mentioned experiments on the two groups C and D, and, calculated all the probabilities given in (33), we may finally decide if the obtained results are in accord with the (33) or with the (34). We have not possible alternatives. If the (34) is confirmed, there is no doubt that mental states follow classical probability regime. If instead the experimental results confirm the (35), we have to conclude that they follow quantum mechanics. So, as previously said, we have performed a lot of experimental verifications in years (Conte 2008; Conte *et al.*, 2003; 2008a; 2007; 2009a; 2009b; 2008b; 2008c). We have examined a sample of about 250 subjects. Always we have obtained that it is the (34) to be validated and never the (33). Let us specify in detail the nature of the results that we have obtained. We have executed three kinds of experiments. In the first case we used ambiguous figures, as test A and B, and they are given in Figure (4), and thus investing the perceptive-cognitive functions. Without any doubt the (34) has resulted always validated confirming quantum interference at the level of mental states. In 68 subjects we also examined the Stroop effect whose importance is well known. EEG and functional neuroimaging studies of the Stroop effect have consistently revealed activation in the frontal lobe and more specifically in the anterior cingulate cortex and dorsolateral prefrontal cortex, two structures hypothesized to be responsible for conflict monitoring and resolution. Accordingly, patients with frontal lesions obtain lower punctuations in the Stroop test when compared to those with more posterior lesions. However, these frontal regions are not the only ones implicated in the effect. Stroop performance has also been associated with the correct functioning of the hippocampus or posterior brain areas. By using Stroop effect we thus explored high level cognitive functions in brain, and also in this case we obtained confirmation of quantum interference as indicated by the (34). Finally, we studied also the phenomenologies that often are retained anomalies in our cognitive performance. We know that humans don't always take the most rational decisions. There is a lot of such cognitive performance that often have been characterized by the term of anomalies. More technically they are expressions of our wishful thinking in which a subject decides that something is true because he would like it to be true. We selected the "anomaly" of the conjunction fallacy that of course is well known. Also in this case we had confirmation that the (34) hold and that we have quantum interference also when exploring such our cognitive tendency. So, the conclusion seems to be evident. In the cases under our experimentation we have found without exceptions that mental states follow quantum mechanics.



Test B. Are these two equal segments? | Test A. Are these two equal circles?

Figure 4. Ambiguous figure used during experimentation on quantum interference effects.

However, I expect some important criticism on this matter so that I will follow step by step all the comments that could arise.

The first is quantum interference. Please, note carefully that the finality of our experiments was to estimate the quantum interference term but in substance my crucial point is and was another. As I explained previously, if a system is really quantum, it must have the so called quantum wave function ψ. Quantum interference arises from existing wave functions and their behaviours in quantum mechanics. Consequently, if we aim to evidence that in our experiments we examined a quantum behaviour of mental states, we have to show that we reconstructed an existing and corresponding wave function ψ of the explored mental states of the humans under investigation. Here is one of the central points of our experimental work. The question arises because the so called Schrödinger's wave function is not an observable and consequently it cannot be estimated. Be carefully. It cannot be measured in the usual sense of this world but nevertheless, it may be determined provided to follow a given procedure. This was in effect the basic aim in writing my paper in 1983. We may reconstruct the wave function ψ if really existing and to this purpose we may follow just the procedure that I indicated in 1983. I repeat it here for clearness.

I select a set of observables, that, just to maintain the correlation with our previous discussion on our experiments, I call A, and I perform the measurement of such observable. I will obtain a statistical set of possible results that I call A_n $(n = 1,2,3,.....)$. I will obtain also the corresponding probabilities and thus I will determine the absolute values $|a_n|$ of the coefficients of the decomposed Schrödinger ψ function

$$\psi = \sum_n a_n \psi_{A_n} \quad (35)$$

We have not reconstructed the ψ- function because, in order to determine the ψ completely, we have to determine the most important parameters of quantum mechanics that are the phases. Actually it is

$$a_n = |a_n| e^{i\alpha_n} \quad (36)$$

To reach this objective, I measure now another set of observables that this time I call B. Following the same procedure of the previous case I will obtain this time

$$\psi = \sum_m b_m \psi_{B_m} \quad (37)$$

We introduce the decomposition

$$\psi_{B_m} = \sum_n c_{mn} \psi_{A_n} \quad (38)$$

and we obtain the final and decisive equations

$$a_n = \sum_m b_m c_{mn} \quad (39)$$

Here is the reason because the (39) is so important. We have here a set of, say, N complex equations and this is to say that we have a set of $2N$ real equations. The basic problem for us is represented from the determination of the phases α_n and β_m. If the (39) enable us to obtain an unique determination of such phases, we may conclude with certainty that we have measured a real quantum system. If the (39) are not soluble for such phases, this means that we have measured a mixture and this is to say that we have examined a classical not quantum system. If, finally, we obtain that the (39) does not permit an unique calculation of the phases, we have to conclude that the measured A and B result dependent one from each other and thus that we have to change the selected set of observables.

In conclusion, the (39) is conceived in so manner that if we arrive to a unique determination of the phases, we cannot have doubt: we are in presence of a real quantum system. And consequently we estimate also the interference term. Therefore, the first problem is to reconstruct, if existing, the possible wave function and this was precisely the result that we reached in our experimental paper of 2009 on NeuroQuantology. In the case of our experiments we may perform all the calculations, and we have that

$$a_1 = b_1 c_{11} + b_2 c_{21}$$
$$a_2 = b_1 c_{12} + b_2 c_{22} \quad (40)$$

Call

$$c_{11} = \sqrt{p(A=+1/B=+1)} = \rho_1$$
$$c_{21} = \sqrt{p(A=+1/B=-1)} = \rho_2$$
$$c_{12} = \sqrt{p(A=-1/B=+1)} = \rho_3$$
$$c_{22} = \sqrt{p(A=-1/B=-1)} = \rho_4 \quad (41)$$

$$a_1 = \rho_5 e^{i\phi_1}$$
$$a_2 = \rho_6 e^{i\phi_2}$$
$$b_1 = |b_1| = \rho_7$$
$$b_2 = \rho_8 e^{i\omega} \quad (42)$$

Note that we have not taken in consideration the phases for c_{ij}.

Let us now apply the (39). We obtain that

$$\rho_5 e^{i\phi_1} = \rho_7\rho_1 + \rho_2\rho_8 e^{i\omega}$$

Solving we obtain

$$\cos\omega = \frac{\rho_5^2 - \rho_2^2\rho_8^2 - \rho_1^2\rho_7^2}{2\rho_1\rho_2\rho_7\rho_8} \; ; \; sen\phi_1 = \frac{\rho_2\rho_8}{\rho_5} sen\omega \qquad \textbf{(43)}$$

Note that, explicating the (43), we obtain

$$\cos\omega = \frac{p(A=+1) - p(A=+1/B=-1)p(B=-1) - p(A=+1/B=+1)p(B=+1)}{2\sqrt{p(B=+1)p(B=-1)p(A=+1/B=+1)p(A=+1/B=-1)}} \qquad \textbf{(44)}$$

that is just the formula for quantum interference that we introduced in the (34).

Let us now apply again the (39). This time we obtain

$$\rho_6 e^{i\phi_2} = \rho_3\rho_7 + \rho_4\rho_8 e^{i\omega}$$

Solving we obtain

$$\cos\omega = \frac{\rho_6^2 - \rho_4^2\rho_8^2 - \rho_3^2\rho_7^2}{2\rho_3\rho_4\rho_7\rho_8} \; ; \; sen\phi_2 = \frac{\rho_4\rho_8}{\rho_6} sen\omega \qquad \textbf{(45)}$$

Note that explicating the (45), this time we obtain that

$$\cos\omega = \frac{p(A=-1) - p(A=-1/B=-1)p(B=-1) - p(A=-1/B=+1)p(B=+1)}{2\sqrt{p(B=+1)p(B=-1)p(A=-1/B=-1)p(A=-1/B=+1)}} \qquad \textbf{(46)}$$

that is still the formula of quantum interference. The probabilities represent the experimental values that we obtained in the experiments. Inserting such values in the (43 and 45), we calculate, if exist, the phases ϕ_1, ϕ_2, and ω, and, finally we may estimate the quantum interference term. In this manner we reconstruct the existing quantum wave function of the mental states and the quantum interference:

$$\psi(A=+) = \rho_5 e^{i\phi_1} \; ; \psi(A=-) = \rho_6 e^{i\phi_2} \; ;$$
$$\psi(B=+) = \rho_7 \; ; \psi(B=-) = \rho_8 e^{i\omega}$$

where, by using the experimental data we have calculated $\cos\omega$ from the (43), and thus we have estimated $sen(\phi_1)$ from the (43) and thus $sen(\phi_2)$ from the (45).

I may take now a further step on. Someone could object as example that in principle we can obtain interference also in the classical wave mechanics, and thus, evaluating in this manner, I should not be so convincing to have indicated that mental states follow quantum mechanics. As answer I evaluate that existing interference in classical mechanics is certainly true. However, the essence of quantum interference is that it exists for discrete observables, detectors, clicks, and it is hard to find such example in classical physics.

There is still a conceptual comment that seems of relevance for me. Quantum mechanics runs about the superposition principle of states that we have repeatedly used in the present formulation. In an elementary exposition it may be re-conducted to two basic statements. If the system is in states that may be described by the quantum wave functions ψ_1 and ψ_2, it may be also in states constituted by ψ_1 and ψ_2, according to the linear transformation

$$\psi = a_1\psi_1 + a_2\psi_2 \, . \qquad \textbf{(47)}$$

Still, if a wave function is multiplied by a complex number different from zero, the new wave function will represent the same quantum state.

The thing that we intend to outline here with careful consideration is that the quantum superposition of states is different substantially from superposition of oscillations in classical physics in classical physics we may have superposition with greater or less amplitude. We have also states in which the amplitude is everywhere equal to zero. In quantum mechanics, instead, the nullity of the wave function simply corresponds to such non existing state.

I think to have introduced satisfactory comments also on this point. Let us examine now the final question that in my opinion could be raised and in particular from the psychologists. They could introduce the following observation. One feature of the experiment is that with one group of subjects, we test B and a brief time later we test for A. With another group of subjects we test only for A. Then we analyse the obtained statistical results inserting them in formulas in order to identify if existing or not quantum interference that is at basis of quantum mechanics.

It could be instead that what we are really testing, is a correlation between the results of B followed by A. In substance, we could have a classical correlation. The reason is that if the human mind first has to solve one task of pattern recognition and then has to solve a similar task of pattern recognition only a short time later, the experience of the first task could influence how the mind approaches the solution of the second task. There are so many mental processes going on subconsciously, which are very likely just classical computation, that the results of the experiments are a consequence of these classical computational processes, and have nothing to do with quantum mechanics and quantum evolution.

This objection is so serious that it deserves careful consideration. Let us me add some comments.

1) I introduced the (35-40) in 1983 just to avoid the risk previously mentioned. Remember that our starting point is extremely clear. If we have a real quantum system, we must arrive to reconstruct uniquely the existing phases of the quantum wave function. When we arrived to write the (39) and thus explicitly the (40) relating directly our experiment, we also were induced to conclude for three and only three possible cases. Or we arrive to an unique determination of the phases (real quantum mechanical case), or we are in the impossible condition to determine the phases (classical case) or we are in the case of an impossible unique determination of the phases and this means that we selected B and A as dependent and thus correlated. Without exceptions the results of our experiments always lead to a reconstruction of the quantum wave function, thus to a unique determination of the phases, and this should exclude the possible case of dependence between B and A.

2) This is an argument that is rather convincing under the profile of quantum theory but the psychologist has all the right to advance his detailed knowledge and thus prospect his counter example. Let us examine such possible objection. In order to show that the results of the experiments can also be explained in a classical manner, one may consider in detail the following model. To avoid confusion by the symbolism let us invert the order of the tests A and B. Reason in the following manner. Let us assume that a person who does the test A has a tendency to remember the colour of the pattern he recognized and will subconsciously try to look for the pattern with the same colour in the test B which followed after a brief time later. This will be a kind of mental influence of the result A on the result B.

For instance, test A resulted in the white pattern (A=+). When the person does test B, some little time later, he will subconsciously also try to identify a white pattern (B=+). But it could also be the other way around. If he recognized the white pattern in test A (A=+) he may have a tendency to recognize more easily the black pattern in the subsequent test B (B=-). One may writes this in the general formula:

$$p(B=+/A=+)=p(B=+)+R\,p(B=-) \qquad \textbf{(48)}$$

where R may be considered to be a correlation parameter varying between -1 and 1, which allows to go from correlation to anti correlation, but also to no correlation when $R=0$. Similarly, one may write that

$$p(B=-/A=-)=p(B=-)+S\,p(B=+) \qquad \textbf{(49)}$$

where S is still a correlation parameter ranging between -1 and 1, and also with possible value $S=0$.

Of course, the conditional probabilities $p(B=-/A=+)$ and $p(B=+/A=-)$ are the complementary of the previous ones. By this classical model one may reproduce our results estimating a value for R and one for S. This is a possible classical model. It was suggested to me from a dear colleague (private communications).

However, let us look carefully to it. Let us examine in detail the model. Without loss of generality, assume

$$p(A=+)=\cos^2\alpha \quad , \quad p(A=-)=sen^2\alpha\,,$$
$$p(B=+)=\cos^2\vartheta\,,\ p(B=-)=sen^2\vartheta$$

The proposed classical model is the following:

$$p(B=+/A=+)=p(B=+)$$
$$+R(1-p(B=+))=p(B=+)+Rp(B=-)$$

$$p(B=-/A=-)=p(B=-)$$
$$+S(1-p(B=-))=p(B=-)+Sp(B=+)$$

$$p(B=-/A=+)=1-(p(B=+)+Rp(B=-))$$
$$p(B=+/A=-)=1-(p(B=-)+Sp(B=+)) \quad \textbf{(50)}$$

Now let us estimate that they must simultaneously hold. So we have that

$$p(+/+)+p(+/-)=1-M\cos^2\vartheta+Nsen^2\vartheta$$
$$p(-/+)+p(-/-)=1+M\cos^2\vartheta-Nsen^2\vartheta$$
$$p(+/+)+p(-/+)=1$$
$$p(+/-)+p(-/-)=1 \quad \textbf{(51)}$$

where $M=S-1$ and $N=R-1$.

The objection that we may move is that it does not result fully consistent. Just to give an example: the fourth equation results equal to the sum of the first two minus the third. Therefore, also in this case in my opinion we do not reach a fully effective classical counter example. We could continue to develop such argument introducing still some more technical comments relating in detail quantum principles and the experimental conditions in which we investigated. However, I see that all such final comments may be overcome also by the direct knowledge of the reader so that, for brevity, I avoid to comment also them. Therefore I arrive to introduce the evidence n. 5

Evidence n.5

By using appropriate tests we arrive to reconstruct quantum wave function and to estimate quantum interference of mental states relating both our perceptive-cognitive functions as well as cognitive functions only.

3. Further Advances

I may now attempt to reach the final conclusion of such a so long presentation. I limit my considerations to the experimental results discussed in the previous section. We have obtained a number of experimental results that seem to move all in the same direction. We have five evidences indicating that quantum mechanics is directly involved in the dynamics of the mental states. Are such experimental results sufficient to establish in a final form that consequently we think in a quantum probabilistic manner?

This kind of answer cannot be accepted as definitively satisfactory. The theme is so complex and articulated that also encouraging results do not authorize in any manner this definitive conclusion. I may say that the experiments give evidences about the role of this theory in our mind dynamics but I think that we cannot go on so much longer such threshold.

Here basic science and psychology may open wide. I elaborate here under the perspective of science. It never accepts results as conclusive and definitive. It only may accept results as established, and this situation happens only when it reaches conceptual and scientific foundations that it must have in its hands. Let us make an example in order to further elucidate my position. Quantum mechanics was introduced from its founding fathers as Bohr, Heisenberg, Jordan, and Pauli, just to quote some authors only. The starting motivation was to study the atoms. Why should this physics pertain to our mind dynamics if it was introduced to study the microphysical level of our reality? It is certainly true that in following years quantum mechanics has been found to have a role also in studies of macroscopic objects.

Andrei Khrennikov has recently written an excellent book (Khrennikov, 2009) that we have also quoted previously. It has a significant title: Ubiquitous Quantum Structure. I invite the reader to read it because it is illuminating under such profile. We could list more and more fields in which the profile of quantum mechanics seems to arise. But in any case the problem remains in our theme. Quantum mechanics is by this time more than eighty years old. It started and still continues to throw a dramatic mess in the basic apparatus of our traditional reasoning: irreducible, ontic indetermination, quantum probability fields, superposition of states, quantum interference, quantum entanglement, quantum tunneling, quantum collapse, are only some of the extraordinary conceptual features that this theory introduces, and it strongly indicates that they are at the basic foundation of our reality.

So again. Why should quantum mechanics have a role in our mind dynamics? What should be the link between quantum mechanics and mind? Why should we accept to consider a theoretical body of physics as entering strongly in the description of mind processes when years and years of studied have clearly indicated that mind is an abstract entity escaping possibly to any tentative to give it a formal physical support and justification?

It is certainly true that from its advent quantum mechanics accustomed us to look at our reality in a completely new manner respect to traditional approach to reality as described by classical physics. As just said, Heisenberg indetermination principle, irreducible indeterminism, the principle of the superposition of states, quantum interference, the question of the quantum measurement, the quantum entanglement are only few examples of the new and upsetting scheme of reality that quantum mechanics points out. Why all this new approach should relate directly our mind dynamics?

It is so hard to answer to a so complex question. But fortunately science does not ask to find an answer to such problem immediately and in a definitive manner. Science strongly demands another thing. In order to accept quantum mechanics having a role in mental dynamics, it demands to find a profound and well fixed conceptual foundation convincing ourselves that quantum mechanics relates mind phenomenology. Otherwise, in absence of such discovered foundations, the problem remains suspended because it has not a theoretical and conceptual support, and we may only accumulate quantum evidences and no more.

This is the direction about which I move our argumentation. To this purpose, there is an excellent phrase of Davis-Hersh and repeatedly quoted also by Mumford those lights the way we have to go along if our aim is to give actual advance about this problem. It states that the study of mental objects with reproducible properties is called mathematics.

Therefore, we need mathematics. We need to give proof of theorems if we actually intend to give support to the problem and this is precisely the way we have to pursue here. Let us sketch the problem briefly.

The question arises with the well known problem of quantum measurement. Suppose we have an instrument designed to measure a dynamical variable, call it A, belonging to some quantum given system S. If S is initially in an eigenstate of A, then the pointer of the instrument will show the corresponding eigenvalue on it. Now, the rather unusual problem arises instead if S is initially in the so called quantum superposition of two quantum states. In this case the pointer will indicate either the value, say a_i, or a_j with probabilities. It will not be partly at a_i, and partly at a_j, even if the initial sate is a superposition of the two quantum states. Therefore the superposition principle is violated in a measurement. At this stage the problem becomes very complex. The theory divaricates. In brief, in quantum mechanics we have a type of time quantum evolution that is causal and it is described by Schrödinger equation. But this theory is not sufficient to describe what happens during a measurement with the above example. We have a second type of time quantum evolution that this time is non causal and that is due to the casual change during a measurement. This second mechanism is the mystery of quantum mechanics. It is often called the collapse of the wave function or wave function reduction just to mention two most used terms to represent this process. Where is the problem? It is that no one knows how it is the manner in which such mechanism is realized in Nature. The actual situation is that the previously mentioned violation of superposition principle in a quantum measurement led von Neumann in 1935 to postulate that we have two fundamentally different types of evolution for a quantum system, the first, as said, is the causal Schrödinger evolution holding in absence of measurement and the second, the collapse of the wave function, that is the non causal change of the wave function during a quantum measurement.

Note the fundamental term about which runs our reasoning. Von Neumann introduced an ad hoc *postulate about the happening of wave function collapse,* and in fact quantum physicists mention quantum mechanics speaking of von Neumann postulate on quantum measurement. Obviously the presence of a postulate in absence of an exact understanding and explanation of the mechanism of wave function collapse has determined a long debate about it, starting with von Neumann and considering other celebrated fathers of quantum mechanics. As example von Neumann and the same Wigner, just to quote two of several and several authors, postulated that it is the advent of consciousness of the observer to determine the collapse of wave function. By contrast, other physicists observed that physics does not need the magical properties of mind, and that we cannot pursue such way to explain how and if the collapse of wave function is induced. Their position is well clear: physics must not require the magical properties of mind to describe the independent observed reality. Some authors questioned that wave function reduction is realized during a quantum measurement owing to the interaction between the microscopic system that is observed and the macroscopic system represented from the device that performs the measurement. Other physicists even denied and continue to deny the same existence of wave function collapse. In conclusion, the question of the wave function collapse has remained a mystery for quantum mechanics. Obviously, the great mental reservation of physicists runs about the basic fact that von Neumann introduced a postulate on quantum measurement. We repeat. He introduced an ad hoc postulate from the outside to the theory just to give justification of the actual reading that we perform on an instrument about the obtained value during a measurement. Science never authorizes theories that have not self-consistence and experimental confirmation of its predictions. The basic fuzzy was that quantum mechanics always resulted confirmed at experimental level but it forced us to live together with such an ad hoc postulate. Here is the basic question about quantum mechanics that was seen often as a non self-consistent theory requiring in fact the addition of an ad hoc postulate, attached from the outside to a theory, in order to explain the physical situation that, during a measurement, we establish a well defined value on the pointer of the instrument but we ignore the manner in which the quantum system does such transition reducing or collapsing its wave function. It cannot escape to the consideration of the reader that such missing self-consistency was forced to live together with the conceptual difficulties arising from the very upsetting ontic foundations of the theory since they in fact prospect a very unusual scheme of reality as previously illustrated.

As a result, one thing is to accept quantum mechanics living with an attached postulate. Another important thing should be instead if we could arrive to give proof of such postulate. It is evident that in this case all the old and new debate about quantum wave function collapse should assume a new outline. According to the phrase *that the study of mental objects with reproducible properties is called mathematics,* in this case a proof should give a new fundamental light about wave function reduction happening.

The approach that I have performed in the last few years has moved in this direction. As repeatedly evidenced in the course of the present paper, from years I have chosen to abandon the traditional scheme of quantum mechanics pursuing a different planning. I have chosen to use the so called Clifford algebra. By using this algebra I have arrived to realize what in this paper I have often called a rough bare bone skeleton of quantum mechanics. A bare bone skeleton of quantum mechanics was an excellent indication that Jordan used years and years ago about a scheme of quantum mechanics that he realized in matrix form (Jordan, 1985). Such my bare bone skeleton of quantum mechanics, realized by Clifford algebra, contains all the basic foundations of quantum mechanics and thus it represents a very good platform for analysis of quantum mechanics. This is the conceptual strategy that I have followed in my studies in my years of activity. Methodologically I use such strategy since I am convinced about the basic statement that the study of mental objects with reproducible properties is called mathematics**.** In order to approach some initial notion on Clifford algebra the reader may give a rapid look to my papers or to the lot of publications existing on this subject, and for the realization of such rough scheme of quantum mechanics the reader may as example examine the contents of my paper recently published on International Journal of Theoretical Physics (Conte, 2000; 2010; Conte *et al.,* 2006).

Now, what is the advance that I retain to have realized by such studies? Just recalling still the great importance of mathematics and its implications at cognitive level, I have given proof of two theorems in Clifford algebra. The first theorem relates the existing Clifford algebra that I have called $A(S_i)$ that is the well known Clifford algebra of spin Pauli matrix. The reader with specialized competence in quantum mechanics knows that it is sovereign in quantum mechanics. The second theorem relates another existing Clifford algebra that I have called N_i , and that it is well known from the algebrists and it is called the dihedral Clifford algebra. Both such algebras are well known from the scholars of Clifford algebra. The importance that I reach by giving proof of such two theorems is that I evidence that such two algebras are strongly linked and that the N_i algebra is obtained when we attribute a precise numerical value to a basic element in the given $A(S_i)$ algebra. Now, considering the thing under the physical profile, all we know what means to attribute a numerical value to a given dynamic variable. Particularly in the language of quantum mechanics this means that we perform a quantum measurement of such variable and we obtain a result reading it on the laboratory instrument that we are using. Consequently we have a new statement.

Statement n. 1
The new result is that I have found that the $A(S_i)$ algebra describes quantum systems when quantum measurements are not performed. In other terms, it describes the standard quantum mechanics. The N_i algebra instead describes quantum systems when we perform a quantum measurement. The first algebra, the $A(S_i)$ algebra, refers to the representation of a particular situation in quantum mechanics where the observer has not been called to measure and to decide as example on the state of a given two-state system. So, it relates the standard quantum mechanics. Through an operation that mathematically is represented by the N_i algebra, the observer finally decides to perform a quantum measurement and to specify which state is the one that will be or is being observed. In conclusion, when it happens the so called wave function reduction or collapse of wave function, we have a transition from the $A(S_i)$ algebra to the N_i algebra .

Statement n. 2
In this manner we give a mathematical description of the so called wave function collapse, supported this time no more from a postulate, that one introduced ad hoc by von Neumann, but from two shown theorems. The interested reader may look in detail to my papers published on this subject. He may verify that we apply such new criteria of transition from $A(S_i)$ to N_i algebra to a number of cases of physical interest in quantum mechanics and we regularly arrive each time to describe the collapse as we always wished to obtain from its starting for quantum mechanics.

Statement n. 3
Still, there is another feature of particular importance. Each time we obtain the results in perfect accord with the von Neumann postulate on quantum measurement.

Statement n. 4
In conclusion, I obtain that when it happens the so called wave function reduction or collapse of wave function, we have a transition from the $A(S_i)$ algebra to the N_i algebra, and still we obtain that all the results are in agreement with von Neumann postulate. Therefore, we arrive to give a complete justification of such postulate. We give its reformulation no more as postulate but this time supported from two well fixed theorems. I say that in this manner we pass from the regime of a postulate to that one of a proof owing to the two shown theorems. In some sense we pass from a physical content given as postulate to a physical content now given and demonstrated by the two existing theorems. In addition, it results that the two algebras, the $A(S_i)$ and the N_i, are interlinked so that by these two theorems we give back to quantum mechanics the self- consistence that always was questioned from its starting of this theory as missing.

In fact, I remember here that von Neumann postulate was attached ad hoc to the theory while instead now the coexistence of such two algebras eliminates any possibility of doubt about this feature of such theory. In conclusion, my view is that we have reached now a very strong, unquestionable, support to the thesis of the existing wave function collapse since now we have two basic theorems at its foundation. I aim that the reader will take in particular consideration a phrase that I am using in the previous statements. I repeat here this phrase that I just used: *through an operation that mathematically is represented by the* N_i *algebra, the human subject (the observer) finally decides to perform a quantum measurement and to specify which state is the one that will be or is being observed. The phrase here is operation represented by the* N_i *algebra. It gives basic support to the thesis of wave function reduction.*

Someone could correctly object, however, that we are not explaining definitively the manner in which such collapse in actual fact happens in physical terms. We are only acquiring knowledge on the operation and on the manner in which it is represented! Instead let us take a step on, and examine the content of such results. I intend to explain better the meaning of the phrase that I use here: *an operation that mathematically is represented by the* N_i *algebra*.

In order to reach this objective we must start from an observation. As we said previously, several solutions were and continue to be proposed every day about the problem of quantum measurement. Some of them attempted to modify strongly the foundations of quantum mechanics. I remember here some examples, as hidden variables, non linear Schrödinger equation, many worlds interpretation, spontaneous localization, decoherence theory, and many other formulations that I do not attempt either to quote here only for reasons of brevity. Many of the proposed formulations hold on the basis to consider a quantum measurement as a quantum system-measuring instrument interaction. This of course seems to be the actual status of the matter since we are accustomed to perform our experiments in laboratories where really force the system to be measured to interact with a measuring apparatus and finally read the result of the measurement. However, we have to introduce a further statement.

Statement n. 5

A measurement is not a physical interaction only. It is an operation that mathematically is represented by the N_i *algebra*. It is here the profound reason because quantum measurements and their mathematical formulations are so important not only in physics but also in neuroscience and in psychology. A measurement is a physical interaction between two systems but, in accord in some manner with Schneider (Schneider, 2005), we cannot avoid to add a basic other feature.

Statement n. 6

A measurement is fundamentally an interaction between languages, perception, and cognition. In other terms, we cannot escape to fix one time for all that a measurement is a semantic acts, just using here Schneider words. We specify. This is an operation that mathematically is represented by the N_i *algebra. I state precisely: a measurement is a cognitive act. It does not exist a measurement without a cognitive task. It is not important if we read directly the result of the measurement on the instrument or if instead it is read automatically, it is not important if the measuring apparatus is macroscopic or not, it is fundamentally important to accept that any measurement is conceived at its source on the basis of a cognitive –semantic act. Any measuring instrument is realized at its source so to perform a semantic-cognitive act and without such basic condition we have not a measurement. A measuring device is a structure that is the object of our perception and of our mental operations. I think this is universally accepted. However, we cannot ignore that such operation of measurement cannot run if we have not previously established the mathematical symbols, the semantic and semiotic functions, in brief … the cognitive performance, that enables us subsequently to express the results of the measurement.*

I repeat here again. It is a semantic act and thus an operation that mathematically is represented by the N_i algebra. Here is the importance of the results that we have obtained. I think that the arguments developed by Schneider in relation to this problem are of fundamental interest. In conclusion, a measurement includes from its starting a semantic-semiotic –cognitive task and this is the reason because quantum measurements are so important for the theme that we have here in discussion.

Statement n. 7

In brief, I arrive to conclude that quantum theory includes in itself not only the description of the reality at the microphysical level. We have ubiquitous quantum mechanics, as Khrennikov outlines in his book. It also envelops the cognitive performance that is required to conceive reality. And this is the reason because quantum measurements are so important for neuroscience and psychology. May we give a final and decisive proof of this last statement? Are we in the condition to support this thesis by a precise and rigorous theorem so that it results unquestionable under the mathematical as well logic profile? The answer is positive. Not only we have the previous mentioned theorems but we have also a further proof that I will now expose. As usually, I start again posing the same question that I outlined previously. Thus once again, how is that a physical theory, born to study the properties of atoms, digress so profoundly from such frontier arriving to consider human cognitive features? May we give a further probe of this? May we in some manner legitimate further such advance and may we find a final justification for this unexpected result?

To this purpose let us start considering another fundamental argument. In 1932 von Neumann showed a result that is of crucial importance for us. He showed that the projection operators and, in particular, quantum density matrices can be interpreted as logical statements. Again I will not enter here in the details of projection operators in quantum mechanics for brevity. However, experts in quantum mechanics know exactly what they represent and the other readers will not have a so great difficulty to embrace also such quantum notions. Of course, a datum is for us of fundamental importance for our discussion.

In brief, von Neumann constructed a quantum matrix logic on the basis of quantum mechanics. Any reader who is sympathizing with the ideas that I am exposing in this paper will be glad of such result. As we may see, it returns in quantum mechanics the general scheme that such theory contains the logic and the logic pertains to human cognition. Also if highly promising, also this result, however, cannot be considered so central and determinant for a number of reasons. In order to have confirmation to the central thesis of our paper and with the uprightness that we are demanding to our exposition, we must proof another result. *We have to show that the result that was obtained from von Neumann may be inverted.*

Statement n. 8

In other terms, we must not show that quantum matrix logic may be constructed on the basis of quantum mechanics but exactly the inverted situation. We must show that quantum mechanics may be constructed on the basis of logic. If we arrive to give proof that quantum mechanics derives from logic, I think that we have completed the circle of our reasoning: we have reached the highest possible support of all the theses that we have exposed until here. It will remain very little possibility to contrast our central thesis that we think in a quantum probabilistic manner.

Statement n. 9

In fact, this is the objective that I have reached by my previous papers (Conte, 2010a; 2010b; 2010c; 2010d). Stated that quantum mechanics runs about two basic foundations., the first being the irreducible indeterminism and the second being the quantum interference, starting with our usual basic Clifford elaboration, I have constructed a Clifford logic approach. Then, following the scheme introduced in the first paper on International Journal of Theoretical Physics (Conte, 2010a) and thus using the two theorems relating respectively *the* $A(S_i)$ *and the* N_i *algebras, I have demonstrated that, according to such Clifford algebraic scheme, the origins of the most fundamental quantum phenomena as the indeterminism and the quantum interference, derive not from the traditional physics itself but from the logic.*

In fact, the title of the paper is *On the Logical Origins of Quantum Mechanics Demonstrated By Using Clifford Algebra.* I think that any other comment is unnecessary. The results in both such papers, the first just published on International Journal of Theoretical Physics and the second previously mentioned, give all the necessary support, the required uprightness and the classical unequivocal mathematical warranty to accept the thesis that quantum mechanics does not relate only matter per se but also cognition. Previously we had experimental verifications and quantum evidences. To such previous results I have had now the support of mathematical theorems and uprightness mathematical derivations relating quantum mechanics. So, the formulation no more seems to be suspended in air. It is not a table with a missing leg.

We may still add an example to clear the substance of our last result. Suppose you fix a logical statement. If you give proof that such logical statement gives quantum interference in the same manner as you obtain quantum interference in quantum mechanics when considering instead only matter objects, it remains one and only one conclusion: *We have logical origins of quantum mechanics, and the only admissible consequent conclusion is that quantum mechanics relates conceptual entities and that we think in a quantum probabilistic manner.* We have given here a rough representation of our results but we hope it will be useful to scholars that are more specialized in psychology rather than in quantum physics. I do not add further evidences here. In my opinion, it is now the exactitude of the full scientific nature that closes the circle of our dissertation.

I may add only some final comments. A remark still arises by my formulation. I have shown in my last papers, using the theorems introduced in the first paper, the logical origins of quantum mechanics. Of course I have to outline here with greatest emphasis that the excellent logic Yuri Orlov, starting with 1977 and when he was in prison Camp 37-2 in Urals in USSR as dissident, started to study this problem. He introduced a so called Wave Calculus based upon Wave logic. He did not use the Clifford algebra but arrived to similar conclusions on the logical origins of quantum mechanics (Orlov, 1978). I invite the reader to read all the papers of this author. They are enlightening about our question.

If we have logical origins of quantum mechanics as consequence we have a logical relativism in this theory. How is that we have not such logical relativism in classical physics? What is the reason because we have instead such strong constraint in quantum mechanics? We give here an answer that of course is in accord with Orlov. The explanation is as it follows. This is an important thesis that arises, and I invite the psychologists to think deeply about it.

Statement n. 10

There are stages of our reality in which it results impossible to unconditionally defining the truth. Logic, language and thus cognition enter with a so fundamental role in quantum mechanics because there are levels of our reality in which the fundamental features of cognition and thus of logic and language, and thus the conceptual entities, acquire the same importance as the features of what is being described. At this level of reality we no more may separate the features of matter per se from the features of the cognition, of the logic and of the language that we use to describe it. Conceptual entities non more are separated from the object of cognitive performance.

Statement n. 11

As correctly Yuri Orlov outlined in his several papers, the truths of logical statements about dynamic variables relating matter structure become dynamic variables themselves in quantum mechanics, and thus the cognition becomes in itself an immanent feature that operates symbiotically with the matter phenomenology that traditional physics aims to represent.

Statement n. 12
This is the profound reason because we started such article affirming that quantum mechanics is the first "physical theory" of cognition. It enables us to discover the first and fundamental principle that interfaces mind and matter.

There are levels of reality in which we no more may separate the features of matter per se from the features of the cognition, of the logic and of the language that we use to describe it. This is the basic reason because we think in a quantum probabilistic manner and this is the reason because quantum mechanics is so important in neuroscience and psychology.

Let me add still a final consideration. The statements that I have enunciated hold on the basic result that there are stages of our reality in which it results impossible to unconditionally defining the truth. We have previously discussed this features related to quantum mechanics. The assertion that there are stages of our reality in which it results impossible to unconditionally defining the truth, may seem so hard and so strong to be accepted but, as if by coincidence, it is well clear to the psychologists. In some cases we lose our right to unconditionally defining the truth. We may attempt to explain it with an example. The reasoning in logic House is that one that usually is identified as abduction.

All men are mortal

Socrates is mortal.

Then, Socrates is a man.

As it is easily observed, it is not said that Socrates is a man. This is the hypothetical character of the abduction and of the reasoning that aims to formulate plausible explanations respect to a given phenomenon. A characteristic of this kind of reasoning is that premises may be of various kinds in the sense that they may incorporate symptoms or signs of different origin. Signs may arise from our perceptive context or from emotions or, still, from our inclinations and rules. As example, wishful thinking arises here. We have:

All men are mortal

Socrates is mortal

I wish that Socrates is a man.

Then, Socrates is a man.

In this case the valuable information enables to admit that Socrates is mortal. Socrates could be a lion or a cat or an elephant. The difference is in the fact that our inclination to think that Socrates is a man plays a decisive role in reaching the final conclusion that Socrates is a man. Now, the basic link with the previously enunciated principle is that in conclusion our subjective inclination and plausibility, introduced in the preliminary section of this paper, pushes us to formulate an hypothesis that of course may be also correct, but on the other hand it is based on a sign that is our inclination, and this inclination does not relate in an absolute manner only the external world but also and fundamentally ourselves. Our subjective wish explains what we think and not what it could be. A "physical theory" of cognition must take into account such our attitude as actually quantum mechanics does as demonstrated by our experiments.

In this manner, by using the previous theorems that we have demonstrated, we have reached some final conclusions:

1) The logical origins of quantum mechanics. This is to say that, as known, quantum mechanics runs about two basic foundations: an irreducible indeterminism and quantum interference. The origins of such quantum fundamental phenomena (irreducible indeterminism and quantum interference) do not lie in physics itself but in the logic. Quantum mechanics relates conceptual entities. We have here a profound link with human cognition considering in particular the fundamental task that, in accord with Schneider; we considered that a quantum measurement must be considered before of all a semantic act.

2) As correctly Yuri Orlov outlined years ago, in quantum mechanics the truths of logical statements about dynamic variables relating matter structure become dynamic variables themselves, and thus the cognition becomes in itself an immanent

features that operates symbiotically with the matter phenomenology that traditional physics aims to represent. *Conceptual entities non more are separated from the object of cognitive performance*

Physical entities are permanently interfaced with conceptual entities in quantum mechanics. Let me allow using such expression: one entity couples as self-image of the other. This is the profound reason because we started such article affirming that quantum mechanics is the first "physical theory" of cognition. It enables us to discover the first and fundamental principle that interfaces mind and matter. There are levels of reality in which we no more may separate the features of matter per se from the features of the cognition, of the logic and of the language that we use to describe it. This is the basic reason because we think in a quantum probabilistic manner of reasoning and this is the reason because quantum mechanics is so important in neuroscience and psychology. By assuming such two conclusions, have we reached the final conclusion that we think in a quantum probabilistic manner.

We have reached a strong theoretical support by showing such theorems. Quantum evidences accumulated in years of experimental research receive here a particular confirmation, based, I repeat, on a strong theoretical support. Let us remains to give still some further elucidations and justifications. For example, Lines ago we said that in quantum mechanics the truths of logical statements about dynamic variables relating matter structure become dynamic variables themselves. *Conceptual entities non more are separated from the object of cognitive performance*. What should such statement mean? We consequently affirmed that, as consequence of the previous theoretical result, it follows that the cognition becomes in itself an immanent features that operates symbiotically with the matter phenomenology that traditional physics aims to represent. Matter entities and conceptual entities coexist in quantum mechanics.

We have here some other convincing argument about such conclusion. Psychologists well know the concept of self image that we used often in this paper and that in recent years has been used frequently (Zak, 2000a; 2000b) by M. Zak. Let us do a step on. In psychology we have the concept of reflection. Reflection is conceived in psychology as the human ability to assume the position of an object in relation to one's own thoughts. Reflection in psychology language is self-awareness by the interaction with the image of the self.

As a result, not only have we given a strong theoretical support by our results. We have found also a direct and evident correspondence with one of the basic notions of psychology. Is it not so impressive the profound link that we find between quantum mechanics, with the supporting theorems that I have shown, and some foundations of psychology?

Note that we have arrived to the concept of reflection and of self-image that, I repeat, is very fundamental in psychology, and that very recently and repeatedly has been used by Zak. Let us introduce some concepts that in our opinion Zak has explained in an excellent manner.

The first notion that we have to recall here is that one of random walk. The term *random walk* was first introduced by Karl Pearson in 1905. A random walk is a mathematical model of a trajectory that consists of taking successive random steps. This kind of process may be studied mainly in physics and in biology but also in ecology, economics, and computer science to quote only some of the applications. It is a fundamental model for random processes in time. For example, the path traced by a molecule as it travels in a liquid or a gas can be represented as random walks. Random walks have their particular importance in the sphere of the biological dynamics. The reason may be rather evident (Zak, 2000a; 2000b) a biological system is an ensemble of cells linked by an informational network. The flow of information is continuously destroyed or delayed or be incomplete as all we well know. As consequence we have stochasticity, and it may be represented by a controlled random walk. This is the reason because random walk is so important in living dynamics. Now, the basic question outlined by Zak is that we have to distinguish a random walk that has a physical origin, from a random walk that instead has a biological origin. In other terms, we have here a profound distinction between non living and living beings. A biological random walk, as characterized in classical physics, must be non linear. In the general case we have processes converging to a stable state, to states of lower complexity and higher entropy. The evolution of living systems is directed instead toward a higher level of complexity if complexity is associated with a number of structural variations. Here the evolution never dies, it produces new configurations. The evolution is directed against the second law of thermodynamics by giving origin to patterns outside the equilibrium (Zak, 2000a; 2000b).

Let us discuss some mathematics briefly. Consider the model of random walk as it has been introduced by Zak in his excellent papers. We write it in the following manner:

$$x_{t+\tau} = x_t + h\,\mathrm{sgn}(R(\pm 1) + \mu) \qquad \mathbf{(52)}$$

Here h,τ are constants indicating the space along the x and the time steps. $R(\pm 1)$ is a random function taking the values from -1 to $+1$ with equal probability and, finally, μ is what we may call a control parameter under the condition that

$$|\mu| \le 1/2\,. \qquad \mathbf{(53)}$$

Note that the equation (52) describes a motion in the physical space. This motion is irregular so that we may introduce (and it is here the most fundamental concept for the argument) a probability space to characterize it. This equation is well known and it is written in the following manner:

$$f_{t+\tau,x} = pf_{t,x-h} + (1-p)f_{t,x+h} \qquad \mathbf{(54)}$$

where $f_{t+\tau,x} = f(t+\tau,x)$ and $f(x,t)$ represents the probability that the moving particle occupies the position x at the instant t while p represents the transition probability and it is given by

$$p = \frac{1}{2} + \mu \qquad \mathbf{(55)}$$

In conclusion, we have three simple but fundamental equations, the (52, 54 and 55). We may say that around such equations runs the problem that we have under consideration.

I repeat: the (52) is written in the physical space. The (54) is written in the probability space. The nature of such basic distinction must result well clear conceptually. Still, let us follow Zak's argument. We have two cases. The first is that $\mu = \mu(x)$ and thus $p = p(x)$ (56) owing to the (55).

This is the case in which the system that we have in examination, interacts with the external world. This is the case in which the equation (54) converges to a stable stochastic attractor as we have discussed previously. We have not interest for such situation. Consider instead the following Zak's case:

$$\mu = \mu(f) \text{ and thus } p = p(f) \qquad \mathbf{(57)}$$

The equation (54) no more is linear, it becomes non linear and the equation (52) is coupled to the equation (54) by the feedback that is given in the (57). We have here an internal loop. Using the Zak's words, the equation (52) simulates the "motor dynamics", that is the actual motion in the physical space and the equation (54) is associated this time with the mental dynamics (the conceptual entities) describing the information flows in probability space. We again are here in the condition to recall the concept of reflection in psychology. The equation (54) represents the probabilistic image of the dynamical system that is given in (52). Matter and conceptual entities turn again to be interfaced. In this manner the probability space becomes the space of the mental dynamics that is realized by an inner loop and corresponding to the actual motion in physical space as given by the random walk described by the equation (52). The probability space and the actual physical space are interlinked and such probability space becomes the space of the mental dynamics, that is to say the space of the human cognition that we have evoked in this paper from its starting. In other sections we have also shown the possibility of its spontaneous arising. In conclusion, in some manner we re-find the primitive thesis of Margenau, of Eccles, of Walker, of the basic results that we have obtained by our theorems in quantum mechanics. The space of our mental dynamics is a probability space. The concept of reflection and/or self-image as given in psychology is illuminating in this framework. We wish to take a step on and give still an example to illustrate such results. I discuss here an experiment that I introduced in 1981 (Conte, 1981a; 1981b).

A human observer is placed before a measuring apparatus in order to detect a possible signal connected to the incoming of a particle. An experimenter decides in fact at random to send or not to send the particle and at selected times the observer is asked if the signal (connected with the particle) is present or not. Since the experimenter can also send a random noise, the observer has an intrinsic uncertainty and he can mistake the noise-signal for the particle signal or he can fail to detect the particle when it has been actually sent by the experimenter since the random noise can interfere with the signal-particle with optimum recording.

We think in a quantum probabilistic manner. Let us examine as an ideal observer may operate on the basis of his brain functioning. He aims to minimize the probability of error. After various attempts that we assimilate to a random walk, he arrives to fix a threshold value, that we call α_1, so that he answers yes if $x>\alpha_1$ in the measuring apparatus and he says not for $x<\alpha_1$ in the measuring apparatus. The ideal observer performs a random walk and by his final criterion achieves his minimum probability of error.

Let us substitute now the ideal human observer with an automaton. Also this device is able to perform optimally this detection. It fixes an arbitrary threshold value α_0 and it can probe yes if $x_1>\alpha_0$, and not if $x_1<\alpha_0$. If the answer is right, it can use the threshold value for the next reading and, in the case of wrong answer, it can iterate the threshold value α_0 by $(\pm\varepsilon)$. By the iterations also the automaton performs a random walk with well defined transition probabilities $P(\alpha_{n+1}=\alpha_n\pm\varepsilon)$ and $P(\alpha_{n+1}=\alpha_n)$. Note that we are reconstructing step by step the argument that we have previously developed.

In contrast with the ideal human observer, the automaton has not consciousness, he has not cognition about the matter, it does not know what the α_n represent and it is only a matter of probability that it will optimize the threshold value α_1 following step by step a linear procedure of iteration. The ideal observer elaborates about the probability $P_1(x)$ of the signal-noise plus signal particle and about the probability $P_2(x)$ of the signal noise only. He can minimize the probability of error on the basis of the inequalities

$$qP_2(x)-pP_1(x)\gtrless 0 \qquad \textbf{(58)}$$

being p the probability of the particle to be sent from the experimenter ($q=1-p$). He fixes the threshold value by the solution of the equation

$$qP_2(x)-pP_1(x)=0 \qquad \textbf{(59)}$$

and he affirms that the signal particle is present for

$$qP_2(x)-pP_1(x)>0\,. \qquad \textbf{(60)}$$

The ideal observer reasons in a quantum probabilistic manner as well as the experimenter deciding to send his message. All the mechanism of the experiment is regulated by the abstract field or space of the probabilities. However, there is still an extraordinary feature that pertains only to our ability of cognitive performance. The experimenter sending the signal knows that on the other hand he has an ideal observer that attempts to identify the presence of the particle. His aim is to induce the ideal observer in error. On the other hand the ideal observer knows that the sending experimenter has the finality to induce him in error and his attempt is to avoid errors. There is a feedback loop between the two cognitive –decision performances of the two human subjects. We have something that resembles the non linear dynamics previously outlined. In some sense the ideal observer may influence the decision of the experimenter to send or not the random noise in quantum mechanical terms we may say that the ideal observer induces a retro-collapse of the wave function characterizing the sending experimenter.

We have given here a further and clear example of what we have previously intended by the concept of reflection in psychology as the human possibility to take position of an observer in relation to one's own thoughts. Reflection here intended as self-awareness via the interaction with the image of the self. A mechanism that, using Zak's results, has been indicated by motor-mental dynamics and summarized in the (54), with connected the (57).

Let us see now as a random walk may be represented in quantum mechanical terms. This is the last argument. Non scholars in quantum mechanics will apologize me if I use for only this time directly quantum mechanics in this paper. Consider the following matrix in accord to various authors (Kempe, 2003)

$$U_n = \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} \quad \textbf{(61)}$$

where a_n, b_n, c_n, d_n are complex numbers. $n = 1,2,3,.........$. We have infinite unitary matrices. We may consider they are infinite random matrices. The unitarity of U_n gives the following conditions to be respected:

$$|a_n|^2 + |c_n|^2 = 1\,,$$

$$|b_n|^2 + |d_n|^2 = 1\,;$$

$$a_n c_n^* + b_n d_n^* = 0\,;$$

$$c_n = -(a_n d_n - b_n c_n) b_n^*\,;$$

$$d_n = (a_n d_n - b_n c_n) a_n^* \quad \textbf{(62)}$$

According to various authors (Kempe, 2003), the time evolution of a quantum walk is given by U_n. Of course, the reader remembers that we consider a bare bone skeleton of quantum mechanics represented by Clifford algebra. U_n is an element of the Clifford algebra given in $A(S_i)$. It results

$$U_n = (\frac{a_n + d_n}{2}) + (\frac{b_n + c_n}{2}) e_1 + \frac{i(b_n - c_n)}{2} e_2 + (\frac{a_n - d_n}{2}) e_3 \quad \textbf{(63)}$$

The evolution of the quantum walk is given in the following way. Consider a dichotomic variable, which is a variable that may assume only two values. As example or +1 or –1. In the case of a particle moving as example along the x-axis, it is usually assumed to introduce the variable Chirality that has corresponding quantum states $|L\rangle$, $|R\rangle$, in the sense that we assume that the particle moves on the left, on the right. At each time step, if the particle has left chirality, it moves one step on the left, and, if instead it has right chirality, it moves one step on the right. Left and right states may be represented by

$$|L\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |R\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \textbf{(64)}$$

The unitary matrix U_n given in (61) or the Cliffordf algebraic element of $A(Si)$ given in the (63), act on the states given in (64) in the following manner

$$U_n |L\rangle = a_n |L\rangle + c_n |R\rangle \quad \textbf{(65)}$$

where, considering the (26) , $|a_n|^2$ represents the probability for the particle to be on the left at step n, and $|c_n|^2$ the probability to be on the right. We have also

$$U_n |R\rangle = b_n |L\rangle + d_n |R\rangle \quad \textbf{(66)}$$

where, $|b_n|^2$ represents the probability for the particle to be on the left at step n, and $|d_n|^2$ the probability to be on the right.

In substance, we assume that, if the particle starts being in $|L\rangle$, its walk evolves in time step, according to the (65). If instead it starts being in $|R\rangle$, its walk evolves in time step according to the (66).

The aim of our argument is that we intend to estimate the probability that the particle is at location k at a given time, and according to quantum mechanics it is given by the square of the modulus of the quantum state vector at k. In other terms we have to calculate the probability amplitude and it results to be given in the following manner

$$|\psi_k(n+1)\rangle = P|\psi_{k+1}(n)\rangle + Q|\psi_{k-1}(n)\rangle\,. \quad \textbf{(67)}$$

This completes the exposition of quantum walk. Now, the interesting feature for us is represented by the two given expressions of P and Q, respectively. They are given in the following manner

$$P = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix} \quad \textbf{(68)}$$

The salient feature for us is that P and Q are still elements of the Clifford $A(Si)$ algebra that we have considered in our results on IJTP and previously discussed also in this paper. Precisely, we have that in the Clifford algebra $A(Si)$ they may be written in one of the two following forms

$$P = a(\frac{1+e_3}{2}) + b(\frac{e_1 + e_2 i}{2})$$

$$Q = d(\frac{1-e_3}{2}) + c(\frac{e_1 - ie_2}{2}) \quad \textbf{(69)}$$

or

$$P = a(\frac{1+e_3}{2}) + b(\frac{e_1 + ie_2}{2})$$

$$Q = d(\frac{1-e_3}{2}) + c(\frac{e_1 - e_2 i}{2}) \quad \textbf{(70)}$$

If it happens that to e_3 it is attributed the value +1 from (69), following the theorems shown in IJTP, we obtain that we pass from the Clifford algebra $A(Si)$ to the Clifford algebra $N_{i,+1}$ that correctly gives $P = a$ and $Q = 0$ as it is necessary in the (68).

If instead it happens that to e_3 it is attributed the value -1 from (70), following the theorems shown in IJTP, we obtain that we pass from the Clifford algebra $A(Si)$ to the Clifford algebra $N_{i,-1}$ that correctly gives $P = 0$ and $Q = d$ (71) as correctly one expects.

In conclusion we have add such elaboration of quantum walk just to confirm again the correctness of the theoretical results that we exposed in International Journal of Theoretical Physics and that support all the present paper. We have a final observation. The exposed arguments runs about the two basic Clifford algebraic elements that we have given respectively in the (69) and in the (70). Both are marked by the following algebraic elements

$\frac{1+e_3}{2}$ and $\frac{1-e_3}{2}$.

In the corresponding quantum mechanical formulation, they are representative of what von Neumann in 1932 interpreted as logical statements. They are now at the basis of the theorems that we have obtained (Conte, 2010a) and of the demonstration given in (Conte, 2010a; 2010b; 2010c; 2010d) on the logical origins of quantum mechanics. It returns what has been demonstrated previously by us. Quantum mechanics includes conceptual entities as linked to measurable entities of physical or biological interest envisaged in the theory. *I reaffirm so my conclusion that we think in a quantum probabilistic manner.*

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This author has published a lot of papers on this matter. We retain the best thing is to quote his last book suggesting to the reader to consider in detail this book and all the references that such author gives of his papers.

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