The world is not an asymmetric graph

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1. Might the world be a mathematical structure? My question is not: might the world *have* a mathematical structure, but might it be *identical* with one? There are two general ways of interpreting this question. If it is whether, given that mathematical structures are abstract objects, the world is an abstract object, the answer has to be negative. Think of the world as wholly concrete (space, time and the spatio-temporal denizens of the world) or as a mix of the concrete and the abstract (if we include universals, laws, propositions and the like), but whichever of these is the case, the world is not purely abstract, as a formal structure is.

One might claim, however, that the world *is* a structure[1] in the sense that it *instantiates* a structure and is nothing else. In other words, all there is to the existence of the world is its being a case of a structure. Another case of the same structure might be a visual representation of it. If the structure that is the world were simple enough to draw, one might draw it and say, 'This is what the world is'. That is how I understand Randall Dipert's thesis that the world is an asymmetric graph (Dipert 1997). He claims to have proved 'for the first time in the history of philosophy' (1997: 349) that, employing the branch of mathematics known as graph theory, the distinctness of relata can be established 'through relations alone', i.e. that, in a nutshell, it is coherent – and indeed true – to suppose that everything that exists is relational in nature. This means, he contends, that Aristotle was wrong to have denied the possibility of pure relationalism in the description of what there is.[2]

Alexander Bird (2007: Ch. 6) picks up Dipert's theory and uses it to argue, not that the entire world is purely relational in nature, but that at the 'fundamental level', whatever that may be, there is nothing but a world of pure powers, all defined relationally in terms of their stimulus and manifestation properties which are themselves pure powers defined similarly. Bird believes that graph theory provides a convincing response to the objection that a purely relational world generates either a vicious circle or a vicious infinite regress.[3] [Elsewhere (Oderberg, 2011) I argue for the defensibility of the

1 From now on, when I use the term 'structure', I mean 'mathematical structure'.

2 In fact, it is difficult to interpret Aristotle's discussion of relations in *Categories* VII, but it is certain that he believed no primary substance, such as a particular animal or lump of gold, could be purely relational in nature.

3 The regress/circularity objection is stated in Lowe 2006: 138 and Robinson 1982: 114–15. Stephen Barker has recently judged the matter case closed in favour of Bird and Dipert, asserting, with little accompanying explanation: 'I think Bird . . . successfully responds to this objection' (Barker 2009: 243). As noted above, I consider the objection undefeated.

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regress/circularity objection.] This aside, there are independent reasons for thinking that the world is not an asymmetric graph. These reasons apply with equal force to Dipert's general thesis about what the world is and, with some modification and elaboration, to Bird's more limited claim about what there is at the fundamental level (assuming there is one).

2. A graph, as standardly defined (Wilson 1996; Harary 1969; notational variations aside) consists of a set V of vertices and a set E of unordered pairs $\{u,v\}$ of elements of V, called edges. Edges, for representational purposes, are simply lines joining one vertex to another, and the vertices are usually represented by dots or points. So a graph – better, a graph diagram – is the visual representation of an abstract formal structure. Vertices u and v are adjacent just in case they are joined by the same edge e. e and u are incident to each other, as are e and v, and two edges incident to a common vertex are also adjacent. The graph consisting of a single vertex is trivial. If, for edge $e = \{u,v\}$, $u = v$, then the edge is a loop.

The primary division of graphs is into those that are *symmetric* and those that are *asymmetric*, which presupposes the key concept of graph *isomorphism*. Isomorphic graph diagrams share the same structure, and may do so even if, visually, they look quite distinct because of vast differences in the respective spatial orientation of the diagrams' vertices and edges. In addition, the vertices of two graph diagrams may have wholly distinct labels and yet be isomorphic. What is necessary and sufficient for graph diagrams G_1 and G_2 to be isomorphic is that there be a one–one correspondence or bijection between their vertex sets such that vertices u,v in G_1 form an edge if and only if x,w in G_2 do, where the bijection maps u onto x and v onto w. From isomorphism we get the concept of *automorphism*, which is the isomorphism of a graph to itself. Of course every graph is isomorphic to itself, but some are only trivially so. A trivial automorphism is simply the mapping of each vertex of G onto itself, i.e. the identity mapping. A non-trivial automorphism, on the other hand, involves a mapping of at least some vertices of G onto distinct vertices of G that is *edge-preserving*. In other words: for some permutation P of $V(G)$, any pair of vertices $\{u,v\}$ of G is an edge if and only if $(P(u), P(v))$ is also an edge. A simple example is a graph diagram consisting of four vertices and four edges structured thus: $G_{2,2} = \{\{u,v,x,w\}, \{\{u,v\}, \{x,w\}, \{u,x\}, \{v,w\}\}\}$. (Picture a square with its corners as vertices.) A rotation of this graph diagram through 180^0 maps u onto w, w onto u, v onto x, and x onto v, thereby preserving all and only the edges of the original. This is a *non-trivial* automorphism because each vertex is mapped onto a distinct one. Since $G_{2,2}$ has a non-trivial automorphism, it is (in the sense of instantiates) a *symmetric* graph. An *asymmetric* graph, by contrast, has no non-trivial automorphisms, only the trivial one. Apart from the trivial graph consisting of a single vertex, the next simplest asymmetric graphs have six vertices (and there are eight such graphs).

3. This is enough background for us to assess the philosophical work to which Dipert and Bird put asymmetric graphs. All graphs are structures of relations. Graph diagrams, then, are a visual way of representing relata (the vertices) and the relations, if any, between them (lines forming edges). Now the kinds of relationalism espoused by both Dipert and Bird do not involve the absurd proposition that all that exists are relations, as though there could be a relation without any relata. True, Dipert sometimes flirts with this thought, as when he says, for instance, that in a symmetric graph structurally identical vertices 'begin to fade into one another' (1997: 345) and that all physical objects, at whatever level, should be thought of as subgraphs of the world graph (1997: 356). But, try as he might to avoid telling us what the vertices of the world graph actually are, at the end of his paper he muses, in the spirit of panpsychism, that they might be 'pure feelings . . . constituting a distinct thought or object only when connected to other such entities' (1997: 358).

In other words, some account must be given of the vertices if one is to give them a physical interpretation. Bird's idea is that the vertices are pure powers or potencies, wholly defined in terms of their relations to stimulus and manifestation properties which are themselves further powers, related either infinitely or cyclically to yet more stimulus and manifestation properties that are themselves powers, and so on. Looked at intrinsically, each power is qualitatively identical to every other power. There is no intrinsic difference between them. Similarly, in seeking to interpret Dipert's view of what the vertices of the world graph are, we should think of them as qualitatively indistinct individuals, perhaps bare particulars with purely relational definitions. Leaving aside 'pure feelings', whatever they may be, we might usefully think of them as qualitatively identical micro-particles. (Dipert would likely resist this; we could rest with his 'pure feelings' if we liked, but the particle interpretation is far more plausible and comprehensible.) The particles exist – barely – but they have no *identity* apart from each one's unique structural description in terms of its relations to all the others. Following Quine's dictum 'no entity without identity', we can say that the existence of Bird's powers or Dipert's particles entails their having identities; but on Bird's and Dipert's common view, these identities are exhausted by the relations of the powers or particles to all the other powers or particles respectively.

The keystone of their account is that only asymmetric graphs guarantee that each vertex has a *unique* relational definition, in other words unique, wholly relational identity conditions. In a symmetric graph, the vertices do not have such identity conditions: they can be permuted in such a way that two vertices can swap places in the graph whilst maintaining exactly the same relations. Note that 'swap places' is best taken spatio-temporally in Dipert's case: the vertices are most plausibly interpreted as concrete individuals. For Bird, the vertices are best interpreted as types of powers, in which case the idea of swapping places should be taken purely formally: in a symmetric graph, there is some pair of property types that have the same relational definition. As Bird puts it, in a symmetric graph the structure 'fails to determine the identity' of the vertices (2007: 140).

4. Far from being case closed, however, there is an argument to show that the interpretation of the world, or some level of it, as an asymmetric graph has consequences so bizarre as to render it highly unlikely that such an interpretation is correct.[4]

How to destroy the universe without leaving home. Suppose we take the world to be one big asymmetric graph. Now, it is possible to turn any asymmetric graph, finite or infinite,[5] into a symmetric one by removing enough edges (Erdös and Rényi 1963).[6] So, in the physical case, say Dipert's model of wholly relational individuals, were any to go out of existence so would their relations to the remaining individuals to which they were previously related. Hence some edges would go out of existence. If the right number of edges in the right positions went out of existence, the remaining edges would constitute a symmetric graph. But Dipert has shown, and I agree, that it is impossible to read off, from a symmetric graph, at least some of the identities of the vertices. This means that some of the remaining individuals, which *had* had their identities fixed within an asymmetric graph, would no longer have their identities fixed: they would literally lose their identities. Would they still exist? No, if their identity conditions failed to obtain; that is Dipert's point, and why he insists the world cannot be a symmetric graph. So what should we say – that if the asymmetric graph became a symmetric one by the removal of some edges, then the entire universe would disappear? An easier way to destroy the universe is hard to imagine. (Well, Dr Evil would need some detailed knowledge of the world structure in order to tell whether he could in fact destroy the world from his armchair, but the limitation is purely epistemic. Note that even if only some individuals lost their identities – and hence existence – through symmetrization, this would knock out all the rest, as it were, since the other individuals would no longer be related to anything and so *ex hypothesi* would cease to exist.) So should we say that all of the existing individuals in this putative asymmetric graph that is the world exist necessarily? Or that some of them do while others are contingent? Yet surely all individuals in the physical world exist contingently. So Dipert has a dilemma: either insist that all or some individuals exist necessarily, so as to ensure the graph remains asymmetric, that is, does not collapse into a symmetric structure; or else concede that the world can be destroyed by the

4 The regress/circularity objection, in my view, shows that the graph-theoretic interpretation *could* not be correct.

5 Apart from the trivial asymmetric graph consisting of one vertex, which would not be Dipert's or Bird's preferred world structure.

6 In fact, Erdös and Rényi refer to the deletion of certain edges and addition of new ones, but they note that not all asymmetric graphs can be 'symmetrized' by the addition of an edge.

removal of some individuals if the resultant graph is symmetric. Neither horn, to my mind, is remotely palatable.

The same objection applies, with modifications, to Bird's graph-theoretical structure of pure potencies at the fundamental level. On an Aristotelian view of universals as necessarily instantiated, by destroying every instance of a universal I destroy the universal: rid the world of all the red things, and I rid the world of redness. So, to generate a symmetric structure for those universals that are pure potencies, I would have to destroy every instance of enough potencies so that the remaining potencies constituted a symmetric graph. Granted, I could not do it from my armchair, but are we still to believe that the loss of some finite number of potencies would destroy every other potency at the fundamental level? We'd better hope that there are not a mere six potencies, since the loss of any one of them would destroy all the rest. To be sure, there would have to be an unimaginably large number of instances of each of those potencies at the fundamental level in order to generate the entire universe with all its levels and complexity, but the idea that the loss of one potency would mean the extinction of the universe (assuming complete supervenience) should strike us as bizarre.

I said that Dipert might respond that all of the individuals in his graph-theoretic world exist necessarily, which is hard to accept. On the other hand, Bird could adopt a Platonist view of universals, according to which all of the powers exist necessarily, whatever happens to their instances. An *ad hominem* point is that Bird is officially neutral between Aristotelianism and Platonism about universals (2007: 12), though the Platonist view appears more congenial to his position.[7] Even assuming Platonism, however, he still faces a problem analogous to that confronting Dipert: it is not so much a worry about the universe's going out of existence, but about its *grinding to a halt*.

We are familiar with the idea that if a power cannot manifest itself, others might not be able to either. Atoms with nuclei of a certain minimum size can emit alpha particles, which are themselves easily absorbed by a range of material objects. If all the atoms having the power of undergoing alpha decay were to go out of existence, then that power could not manifest itself, and so neither could the power possessed by any remaining objects of absorbing alpha particles. These remaining objects would, of course, still exist, since a material object is not a potentiality but an actuality. We could even accept that their power of absorbing alpha radiation continued to exist but that it would be incapable of manifestation. Now consider the thought that all there is at the fundamental level are pure potencies and their

7 A general problem is that it is unclear how a Platonist interpretation of asymmetric graphs for potencies is supposed to work at the level of physical instantiation. There is no straightforward way in which the asymmetric graph constituting the formal structure of Platonic properties can be instantiated in the physical world, and Bird gives us no idea as to how this might operate.

possessors (or maybe the possessors of potencies just are themselves bundles of further potencies, if this makes any sense). If all of the instances of one kind of those possessors ceased to exist, the powers essential to that kind would not able to manifest themselves. Note that on Platonism, the kind and the powers essential to it *would* still exist even though none of their instances did. This, however, is irrelevant to the present point, which is that since each power is essentially related to its manifestation, and each manifestation is a further power, if one manifestation could not occur then no power could manifest itself – and so the fundamental level would grind to a halt. The necessarily existing powers would all still be relationally defined, to be sure, but none of their *tokens* would be able to manifest themselves if all of the tokens of a single power type ceased to exist. For the tokens whose type was essentially related to this single power type would themselves not be able to manifest themselves, nor would the tokens whose type was essentially related to this second type, and so on. For Bird, the fundamental level is that of properties with 'non-redundant causal powers' (2007: 13) which (together with the objects that possess them – presumably a subset of the objects of physics) generate all of the natural laws upon which supervene all the other objects, properties, and laws found in nature. On his view, then, a consequence of the fundamental level's grinding to a halt is the entire universe's grinding to a halt. But is that what we should expect from the disappearance of all the instances of a single fundamental power? It is no help to reply that the manifestation of a power is merely an unactualized possibility: if the things that would, in appropriate conditions, produce the manifestation ceased to exist, then the powers whose tokens were essentially dependent on those manifestations could not (as a matter of metaphysical necessity) be exercised; so no power could be exercised since they are all essentially related to one another – and the wheels of the universe would stop turning. Unless, that is, one of three possibilities obtains. (i) Every time a stimulus is applied, whatever produces its manifestation miraculously pops back into existence to ensure the power is exercised. But we should not import miracles or unexplained coincidences into the ordinary workings of the universe. (ii) Every time a stimulus is applied, whatever produces its manifestation is *guaranteed* to pop back into existence to ensure the power is exercised. But what underwrites the guarantee? (iii) All powers at the fundamental level are necessarily manifestable, that is, whatever manifests them is guaranteed to exist. But what then happens to contingency? What independent reason could there be for positing these necessary existents? (Maybe the truth of metaphysical atomism, but that's another story.) In short, Bird's reworking of Dipert is no more tolerable than Dipert's own position.

Since there is no entity without identity, nothing that lacks identity conditions can exist. The Destroying the Universe objection has the universe going out of existence when sufficient of its vertices do such that the result is a symmetric graph. Another possibility, however, is that the rest of the world does not go out of existence altogether; rather, *new* entities with different graph-theoretic descriptions, and hence different identity conditions, come into existence. What this presupposes is that there is some sort of guarantee – whence it comes is difficult to ascertain – that the universe will never collapse into a symmetric structure. Suppose this to be the case. Then, the standing possibility is that of ever-new kinds of thing (or power) coming into existence as the world's asymmetric graph-theoretical properties change (due, again, to the contingency of the physical).

Such a world does not seem to be the one of our experience. If Bird is right about the supervenience of everything physical on the fundamental level, we might wonder why new macroscopic kinds are not regularly replacing existing kinds. Yet they are not. If this is because they *cannot*, then whence the guarantee? My objection is not that new things and kinds cannot come into existence at all; of course they do. Rather, the problem is with the way in which this might happen on the graph-theoretic approach. We should, if the approach is correct, expect new things and new kinds (at both the micro- and macro-levels) to be regularly popping into existence due to the shifting properties of the asymmetric world graph (itself a result of the contingency of the physical). Now, things do of course regularly pop into existence, but by normal causal processes. On the graph-theoretic interpretation now under consideration, the disappearance and appearance of new things/kinds of thing would seem to have nothing to do with regular causal processes; they can and should occur at great spatio-temporal distances as well, simply in virtue of one or more vertices' going out of existence somewhere, sometime. One might doubt whether any of the non-local features of quantum theory could explain such radical appearances and disappearances. In any case, a world of what we might call 'constant identity swaps' is not the one of our experience, at either the macro- or micro-levels.

5. The Destroying the Universe objection does not demonstrate that the world could not be an asymmetric graph. But it does show the unacceptable consequences of its being so. Various moves, some suggested above, might be made to block the consequences or palliate their counterintuitiveness. Still, without more and better reasons for thinking the entire graph-theoretical approach to be independently plausible, the risk remains high that such moves would be either ad hoc or no more plausible than the consequences they are designed to repel.[8]

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The world as a graph: defending metaphysical graphical structuralism

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Metaphysical graphical structuralism is the view that at some fundamental level the world is a mathematical graph of nodes and edges. Randall Dipert has advanced a graphical structuralist theory of fundamental particulars and Alexander Bird has advanced a graphical structuralist theory of fundamental properties. David Oderberg has posed a powerful challenge to graphical structuralism: that it entails the absurd inexistence of the world or the absurd cessation of all change. In this article, I defend graphical structuralism. A sharper formulation, some theorems about such structures, and careful attention to the interaction of metaphysical and mathematical features, shows that the absurdities depend on assumptions that are not essential to the view and brings to light a surprising fact about the necessary structure of fundamental properties.

Dipert (1997) proposes that 'the concrete world is a single, large structure induced by a single, two place, symmetric relation... [which is] best analyzed as a certain sort of graph' (1997: 329), namely a graph of nodes and edges,

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