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Quantum Selections, Propensities and the Problem of Measurement

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ABSTRACT

This paper expands on, and provides a qualified defence of, Arthur Fine's selective interactions solution to the measurement problem. Fine's approach must be understood against the background of the insolubility proof of the quantum measurement. I first defend the proof as an appropriate formal representation of the quantum measurement problem. The nature of selective interactions, and more generally selections, is then clarified, and three arguments in their favour are offered. First, selections provide the only known solution to the measurement problem that does not relinquish any of the explicit premises of the insolubility proofs. Second, unlike some no-collapse interpretations of quantum mechanics, selections suffer no difficulties with non-ideal measurements. Third, unlike most collapse interpretations, selections can be independently motivated by an appeal to quantum propensities.

1 Introduction

In a series of papers in the late 1980s, Arthur Fine proposed a novel solution to the quantum measurement problem, in terms of *selective interactions*, or as I shall call them, selections. The reception to Fine's approach has been nearly mute.[1] But in light of recent developments and difficulties with other proposals for solving the quantum measurement problem, it may be worth taking another look at Fine's proposal. In this paper, I expand on Fine's original proposal by providing a general characterisation of selections that is independent of the measurement problem. I then defend selections as a valuable alternative to extant interpretations of quantum mechanics.

My defence of Fine's original proposal is a qualified one. Unlike Fine, I do not tie the concept of a selection to that of a measurement interaction. I also reject Fine's own philosophical defence of selections as measuring 'aspects'. However, suitably re-interpreted as testing quantum dispositions or propensities, selections are coherent; and they have some definite advantages over other widely-discussed options in the interpretation of quantum mechanics.

Although I shall argue that selections are not conceptually linked to the measurement problem, it is easier to introduce them formally in the context of the so-called insolubility proof of the measurement problem. The first part of the paper is devoted to a discussion of this proof. In Section **2** of the paper, I introduce some preliminary distinctions and notation, and I describe the basic intuition underlying the measurement problem. In Section **3**, I describe the premises of the insolubility proof and, in Section **4**, I defend it as an appropriate formal representation of the problem of measurement. In the second part of the paper, I turn to the selections approach. Thus, in Section **5**, I introduce the concept of a quantum selection, and I argue that selections are fully compatible with a) the unitary dynamics of the Schrödinger equation,

[1] The exception is Stairs ([1992]), whose reaction, like mine, was mixed. But my criticisms of Fine's approach are not Stairs'. On the contrary, I believe that the characterization of selections provided in Sections **5**, **6** and **7** dispenses with most of Stair's criticisms.

and b) the denial of the ignorance interpretation of mixtures. I show that selections can solve the measurement problem without relinquishing any of the explicit premises that generate the insolubility proof. In Section **6**, I show that selections have at least one advantage over several no-collapse interpretations: unlike these interpretations, selections can naturally accommodate non-ideal measurements. I then argue, in Section **7**, that the selections approach, unlike the collapse postulate, is not at all ad hoc, but can be motivated independently by an appeal to quantum propensities.

2 The problem of quantum measurement

2.1 The ignorance interpretation of mixtures

In the most general statistical operator formalism of quantum mechanics, systems can be in pure or in mixed states. A pure state is represented by an idempotent operator of trace one; that is, a projection operator $P[\phi]$, upon a particular subspace ϕ of the Hilbert space. By contrast, a mixed state, or a mixture, is a sum of such projectors upon pure states v_i with associated statistical weights (p_i, $0 \leq p_i \leq 1$, with $\Sigma p_i = 1$), represented by a non-idempotent operator of trace one: $W = \Sigma p_i P_{[v_i]}$.

Mixtures come in two varieties, *proper* and *improper*. An improper mixture is the state ascribed to the component of a composite system in an entangled superposition, and results from the application of the axiom of reduction to the composite state. A proper mixture, on the other hand, is not improper, and typically results from a preparation procedure.[2]

A much-discussed interpretation of quantum mixtures is the ignorance interpretation. According to this interpretation, a quantum system is in a mixture $W = \Sigma p_i P_{[v_i]}$ if and only if the system is really in one of the pure states $P_{[v_i]}$, but we do not know which one. Thus, on this interpretation, the probabilities $\{p_i\}$ are subjective and merely reflect our degree of ignorance.

It is an open question whether this interpretation can be applied to proper mixtures; typically, that may only be decided on a case-by-case basis. The following decisive argument shows that the ignorance interpretation is never available for improper mixtures.[3] Consider a composite system S_{1+2} in a pure state $W_{1+2} = P_{[\psi]}$, where $\psi = \Sigma_{i,j} c_{ij} v_i \otimes w_j$, and where v_i, w_j are the eigenstates

[2] The terminology was first introduced by D'Espagnat ([1971], p. 87) and I follow his use precisely.

[3] The original argument can be found in D'Espagnat ([1971], pp. 86–7), although in an incomplete form which assumes that the contradiction arises only when both reduced mixtures are given the ignorance interpretation. Hughes ([1989], pp. 149–51) contains the same incomplete version.

of A, B with corresponding eigenvalues a_i, b_j. The *reduced states* W_1, W_2 can be derived from the standard identifications (expressions *, in Appendix 1). We obtain: $W_1 = \Sigma_i c_{ii} c_{ii}^* v_i v_i^*$, and $W_2 = \Sigma_j c_{jj} c_{jj}^* w_j w_j^*$. W_1, W_2 are improper mixtures, found by derivation from the composite state W_{1+2}. Let us now assume that subsystem S_1 (S_2) is really in one of the states $v_i(w_j)$ with probabilities $|c_{ii}|^2$ ($|c_{jj}|^2$). The state of the combined system can then be reconstructed, in the manner described in Appendix 1. We find that $W_{1+2} = \Sigma_i c_{ii} c_{ii}^* v_i v_i^* \otimes W_2$ (or $W_1 \otimes \Sigma_i c_{jj} c_{jj}^*\ w_j w_j^*$, or if both W_1 and W_2 are given the ignorance interpretation, then: $W_{1+2} = \Sigma_i c_{ii} c_{ii}^* v_i v_i^* \otimes \Sigma_i c_{jj} c_{jj}^* w_j w_j^*$). Thus, on the assumption that W_1 (or W_2, or both) can be given the ignorance interpretation, we find that W_{1+2} is a mixture; but by hypothesis, W_{1+2} is a pure state; therefore by *reductio*, neither W_1 nor W_2 can be given the ignorance interpretation.

The ignorance interpretation is nonetheless often thought to be required for a satisfactory solution of the problem of measurement. For it is often assumed that a solution to the problem of measurement requires that the final state of the measuring device be a pure state, namely an eigenstate of the pointer position observable. One aim of this paper is to show that this assumption is mistaken.

2.2 The eigenstate–eigenvalue link

We can express in this framework the orthodox interpretative principle of quantum mechanics, the *eigenstate–eigenvalue* link. The basic version of this principle is often formulated as follows:

> *basic e/e link*: A system has a value o_1 of a physical property O if and only if the system's state v is an eigenvector of the self-adjoint operator O that represents this physical property (i.e. if $Ov = o_1 v$).

Note that this is a necessary and sufficient condition. In other words, if the system is not in one of the eigenstates of an operator (if for instance the system is in a non-trivial superposition of eigenstates of O such as $c_1 v_1 + c_2 v_2$), then we are not entitled to say that the system has a value of the property represented by the operator in question.

However, in this paper I will (inspired by Fine [1987]) formulate the eigenstate–eigenvalue link in a more developed version, as follows:

> *extended e/e link*: A system has a value of a physical property if and only if the system's state is a) an eigenvector of the operator O that represents this physical property, or b) a proper mixture $W = \Sigma p_n W_n$, where O takes a value with certainty (i.e. with probability one), in every W_n.

The crucial addition of clause b) allows us to ascribe values to the observables of a system in a mixed state, without requiring that the mixture in question be ignorance interpretable. Hence, this formulation allows us to ascribe values to observables of systems even when the systems are not in eigenstates of the corresponding operators. However, the ascription of values is still highly constrained. Values are ascribed to systems whose states are *proper* mixtures are over states W_n only if the relevant observable takes a value with probability one in each W_n. This rules out our ascribing values to improper mixtures such as those that represent the state of each component of an EPR-entangled state, and hence releases us from any undesirable commitment to Bell-like inequalities, or Kochen-Specker proofs. But it also means that O-eigenstates fulfil this condition by definition; thus the name (extended e/e link) is well deserved. There is an important reason why the (extended e/e link) formulation is to be preferred, which I will discuss in due course.

2.3 The quantum theory of measurement

In order to make a measurement, we must let the quantum object interact with a measuring device. The quantum theory of measurement, as first formulated by Von Neumann ([1932]), ascribes a quantum state to the measuring device, and treats the interaction as a quantum interaction, i.e. one that obeys the Schrödinger equation.

The theory further supposes that the observable of the system that we are interested in is represented by self-adjoint operator O, with eigenvectors $\{\phi_n\}$ and eigenvalues $\{\lambda_n\}$. The pointer position observable A is represented by the self-adjoint operator A, which has eigenvectors $\{\gamma_m\}$ with eigenvalues $\{\mu_m\}$. (And let us here further assume that $n = m$, without loss of generality.)

Suppose, then, that we have an object initially in state $W_o = \Sigma_n p_n P_{[\nu_n]}$, where each ν_n may be expressed as a linear combination of eigenstates of the observable O of the system that we are interested in (i.e. $\nu_n = \Sigma c_i \phi_i$); and a measuring device in $W_a = \Sigma_n w_n P_{[\gamma_n]}$. Throughout the paper, I refer to the observable represented by the operator $I \otimes A$, as well as that represented by A, as the pointer position observable. The eigenvalues of this observable are therefore given by the set $\{\mu_n\}$. As the interaction between the object system and the measuring device is governed by the Schrödinger equation, there must exist a unitary operator U that takes the initial state of the composite system (object system + measuring device) into its final state at the completion of the interaction, as follows: $W_o \otimes W_a \rightarrow U(W_o \otimes W_a)U^{-1}$. (For further details of the interaction formalism, see Appendix 1).

We can now state the basic intuition behind the problem of measurement. Take a system in an arbitrary superposition $\nu_n = \Sigma c_i \phi_i$. Then, due to the linearity of the Schrödinger equation, at the conclusion of an ideal measurement interaction with a measurement apparatus in any pure state, the composite (system + device) will be in a superposition of eigenstates of the pointer position observable. And according to either version of the (e/e link), the pointer position observable cannot have a value in this state. But surely quantum measurements do have some outcomes – i.e. they have some outcome or other? Hence the quantum theory of measurement fails to describe quantum measurements completely!

3 The insolubility proof of the quantum measurement

The insolubility proofs are attempts to formally describe the measurement problem, in order to display precisely the set of premises that come into contradiction. The proofs go back to Wigner ([1963]), and include Earman and Shimony ([1968]), Fine ([1970]), Brown ([1986]), and Stein ([1997]).

3.1 Some notation

First let me introduce some notation, following Fine ([1970]). Let us denote by Prob (W, Q) the probability distribution defined by Prob_w $(Q = q_n)$, for all eigenvalues q_n of Q. And let us denote *Q-indistinguishable* states W, W′ as $W \equiv_Q W'$. Two states W, W′ are *Q-indistinguishable* if and only if Prob (W, Q) = Prob (W′, Q).

We may now enunciate the following two conditions on measurement interactions. The insolubility proof (Appendix 2) purports to show that these two conditions are inconsistent with the Schrödinger dynamics (a fourth condition is actually required, as we shall see in Section **4**).

3.2 The transfer of probability condition (TPC)

$$\mathrm{Prob}(U(W_o \otimes W_a)U^{-1}, I \otimes A) = \mathrm{Prob}\ (W_o, O)$$

The transfer of probability condition (TPC) expresses the requirement that the probability distribution over the possible outcomes of the relevant observable O of the object system should be reproduced as the probability distribution over possible outcomes of the pointer position observable in the final state of the composite (object + apparatus) system.[4] (TPC) entails the

[4] (TPC) is essentially equivalent to Busch, Lahti and Mittlestaedt's *probability reproducibility condition* ([1991], p. 32). Busch, Lahti and Mittlestaedt require that the probability distribution for the required observable defined by the initial state of the object system is reproduced in the probability distribution for the pointer observable in the final *reduced state* of the apparatus. Suppose that $2W_a^f$ represents the final reduced state of the apparatus, derived from the final composite state $\hat{U}(W_o \otimes W_a)\hat{U}^{-1}$ by the standard identifications (see expressions (*) in Appendix 1). The Probability Reproducibility condition reads: Prob (W_o, O) = Prob (W_a^f, A) which, given the derivation of the reduced state W_a^f from the final state of the composite by means of (*), is provably equivalent to (TPC) for observable A.

following minimal condition on measurements employed by Fine ([1970]) and Brown ([1986]): a unitary interaction on a (object + apparatus) composite is a W_a measurement only if, provided that the initial apparatus state is W_a, any two initial states of the object system that are O-distinguishable are taken into corresponding final states of the composite that are $(I \otimes A)$-distinguishable. So we can use the pointer position of the measuring apparatus to tell apart two initial states of the object system that differ with respect to the relevant property.[5]

But is (TPC) really a necessary condition on measurements? It could be argued that an interaction that transfers only part of the probability distribution of the object observable to the pointer observable is nonetheless a measurement, albeit only an approximate one, for some information is thereby transferred. (For instance, an interaction that can only distinguish two particular O-eigenstates is a measurement of sorts.) This worry about (TPC) seems deep and legitimate to me. I will argue in Section **4.3** that the measurement problem arises in the highly idealised conditions imposed by the formal quantum theory of measurement; and in the context of such idealisations (TPC) is justified.[6]

3.3 The occurrence of outcomes condition (OOC)

$$U(W_o \otimes W_a)U^{-1} = \Sigma c_n W_n \quad \text{where } \forall W_n \exists\, \mu_n\text{: } \mathrm{Prob}_{W_n}\ (I \otimes A = \mu_n) = 1$$

The occurrence of outcomes condition (OOC) is often taken to express the requirement, inspired by the *eigenstate–eigenvalue link*, that the final state of the composite be a mixture over eigenstates of the pointer position observable. But to be precise, it expresses the more general idea that the final state of the composite must be a mixture over states in each of which the pointer position observable takes one particular value or other with probability one.

I can now provide the main reason for adopting the (extended e/e link) as formulated in the previous section. It is conventional wisdom that a solution to the measurement problem can always be provided if the eigenstate–eigenvalue link is denied, in particular its necessary part.[7] But now note that

[5] Thus Fine and Brown's condition is weaker than (TPC): it does not imply that $(I \otimes A)$-distinguishability entails O-distinguishability. It is only a necessary but not sufficient condition on measurements.

[6] It is in addition important to emphasise that i) selections are not generally committed to (TPC), and ii) even those selections that obey (TPC) are able to account for a very large class of approximate non-ideal measurements. See the discussion in Section **6**.

[7] That is, at any rate, how modal interpretations solve the measurement problem. See, for illustration, the essays in Dieks and Vermaas ([1998]).

(OOC) follows from (extended e/e link), together with the fact that quantum measurements have outcomes (i.e. that they have one particular outcome or other). A stronger condition would follow from (basic e/e link). However (OOC) is strong enough for the insolubility proof: it is possible to escape it, and hence solve the measurement problem, by denying (extended e/e link), and thus denying that (OOC) is required for measurements to have outcomes. Hence (extended e/e link) preserves conventional wisdom while being the weaker condition. This allows us to characterise a wider class of interpretations that are committed to a measurement problem, and a narrower and more precise class of those that are able to evade it by denying the semantic rule for the ascription of values to observables.

4 A defence of the insolubility proof

4.1 Stein's critique

Howard Stein ([1997]) provides an interesting critique of the insolubility proof. He begins by deriving a lemma in the theory of Hilbert spaces that has as a direct application a version of the insolubility proof (for the details, see Appendix 3). This lemma, he argues, is true given the ignorance interpretation of mixtures, but does not necessarily follow without that interpretation. And, he continues, the ignorance interpretation of mixtures presupposes the wrong picture of quantum states. A quantum mixed state represented as a statistical operator is not an ensemble of pure states, but rather an assignment of probabilities to values of dynamical variables, i.e. to observables of the system. Although in some circumstances the ignorance interpretation may be given, it is not generally called for. The statistical operator formalism does not invite the ignorance interpretation and, Stein concludes, the insolubility proof cannot constitute an accurate representation of the measurement problem.

Throughout this paper I will adopt Stein's understanding of quantum states as an assignment of probabilities to the possible values of a system's dynamic quantity. I will refer to it as the *standard understanding of quantum states*, as I believe it to be established in the literature. There are two reasons, however, why I want to resist Stein's conclusion. The first is that the ignorance interpretation of mixtures is not strictly required for the formulation of the insolubility proof: the proof may be a valid representation of the measurement problem even if the ignorance interpretation is not appropriate. The second is that the types of idealisations that go into the formulation of the insolubility proof, which Stein's critique may be taken to question, are also part and parcel of the quantum theory of measurement, within which the measurement problem arises. The insolubility proof captures as much of the measurement problem as there is to be captured.

4.2 Ignorance is not required

First, note, as a preliminary observation, that the insolubility proof can be stated in a manner that respects the standard understanding of quantum states, for the statements of conditions (OOC) and (TPC) given in the previous section are *prima facie* perfectly consistent with that understanding of statistical operators. These conditions are expressed not in terms of the pure states that compose the relevant mixtures, but in terms of probability distributions defined over the possible values.

I claim that the ignorance interpretation is not required for the insolubility proof. (OOC) is a strictly weaker condition on the final state of the composite than the ignorance interpretation. For suppose that the final state of the composite is degenerate; then it possesses no unique representation in terms of pure states. (OOC) is happy to accept this plurality of representations. By contrast, the ignorance interpretation insists that only one among these representations is physically meaningful—one which contains the pure state that the system *really* is in, with the corresponding epistemic probability. But that means that the ignorance interpretation does not and cannot be used to motivate (OOC). Rather, as has already been emphasised, (OOC) is motivated by the (extended e/e link), together with the requirement that quantum measurements have outcomes and the standard understanding of quantum states. Thus rejecting the ignorance interpretation cannot by itself suffice to explain why (OOC) may fail. And it is (OOC), not the ignorance interpretation, that figures as a premise in the insolubility proof.

There are, however, two important caveats to the above argument. The first one is this: it is nonetheless the case that when the final state of the composite is non-degenerate, (OOC) coincides with the ignorance interpretation. In that particular case, there is only one spectral decomposition of the system's mixed state, and it is also true that only for a particular (pure) eigenstate of the observable will the probability of any particular eigenvalue be one. Perhaps this coincidence underlies Stein's thought that the ignorance interpretation is somehow involved. However, this coincidence cannot provide an argument in favour of Stein's conclusion, because in general (OOC) cannot be justified merely by an appeal to the ignorance interpretation.

The second caveat to my argument against Stein's conclusion cannot be dismissed so lightly. It concerns the use in Fine and Brown's insolubility proof of a condition called *Real Unitary Evolution* (Brown [1986]). According to this condition, the unitary evolution of a mixed state is given by the unitary evolution of its component pure states. Suppose that W_o, W_a are the statistical operators representing the initial states of the object system and measuring device respectively. And suppose that $W_o = \Sigma_n c_n\ P_{[\phi_n]}$, and $W_a = \Sigma_m d_m\ P_{[\gamma_m]}$. Brown states the principle of real unitary evolution as follows:

Real Unitary Evolution (RUE):

$$\begin{aligned}\hat{U}_t(W_o \otimes W_a)\hat{U}_t^{-1} &= \hat{U}_t(\Sigma_n c_n\ P_{[\phi_n]} \otimes \Sigma_m d_m\ P_{[\gamma_m]})\hat{U}_t^{-1} \\ &= \Sigma_{n,m} c_n d_m \hat{U}_t (P_{[\phi_n]} \otimes P_{[\gamma_m]})\hat{U}_t^{-1} \\ &= \Sigma_{n,m} w_{nm}\ P_{[\hat{U}_t(\phi_n \otimes \gamma_m)]}\end{aligned}$$

where $w_{n,m} = c_n \times d_m, 0 \leq w_{nm} \leq 1$ for all values of n, m.

The status of (RUE) has been a matter of some debate, but I think everyone would agree that it is motivated by the ignorance interpretation. In introducing it explicitly, Brown wrote:

> It should be clear, moreover, that the principle is an extremely natural extension of the ignorance interpretation of mixtures, which as a rule is postulated for instantaneous ensembles, to the case of ensembles of systems whose states are evolving over time according to the Schrödinger equation. (*Ibid.*, p. 860)

In fact (RUE) is logically entailed by the dynamical extension of the ignorance interpretation. According to the ignorance interpretation, a mixed state represents our subjective degree of ignorance of the (pure) state of a system. Its dynamical extension will then state the following: any dynamical evolution of the system that fails to provide us with additional information about the initial state of the system must result in a final state that reflects our initial uncertainty. In other words, the pure states must evolve unitarily and independently, with coefficients c_n, d_n that are invariant under this evolution—and that is indeed what (RUE) asserts. Conversely, (RUE) imposes exactly the same condition on the time evolution of states that would be expected if the dynamical extension were true. In a formal sense at least, (RUE) is equivalent to the dynamical extension of the ignorance interpretation.

However, it does not seem to have been noticed that the insolubility proof does not employ as strong a condition as (RUE), but rather:

Quasi-Real Unitary Evolution (QRUE):

$$\begin{aligned}\hat{U}_t(W_o \otimes W_a)\hat{U}_t^{-1} &= \hat{U}_t(\Sigma_n c_n\ P_{[\phi_n]} \otimes \Sigma_m d_m\ P_{[\gamma_m]})\hat{U}_t^{-1} \\ &= \Sigma_{n,m} w_{nm}\ \ \hat{U}_t(P_{[\phi_n]} \otimes P_{[\gamma_m]})\hat{U}_t^{-1} \\ &= \Sigma_{n,m} w_{nm}\ P_{[\hat{U}_t(\phi_n \otimes \gamma_m)]}\end{aligned}$$

where $0 \leq w_{nm} \leq 1$ and $w_{nm} = 1$; but w_{nm} need not equal $c_n d_m$.

This condition is strong enough to generate the inconsistency between (OOC), (TPC) and the Schrödinger equation.[8] Crucially, it is not equivalent

[8] I have made the use of (QRUE) explicit in my presentation of the insolubility proof in Appendix 2. Brown ([1986]) made (RUE) explicit, but it is (QRUE) which is implicitly employed in Fine ([1970]). Stein ([1997]) invokes the commutativity between $(I \otimes A)$ and $\hat{U}\ (W_o \otimes W_a)\ \hat{U}^{-1}$ which

to the dynamical extension of the ignorance interpretation. The latter entails (RUE), which is a special case of (QRUE), but the dynamical extension is not entailed by (QRUE). There are possible unitary interactions in which (QRUE) holds but the ignorance interpretation (and (RUE)) are plainly false. For there are possible choices of n, m for which (QRUE) is true, while the ignorance interpretation and (RUE) are not. Thus (QRUE) is neither a natural extension of the ignorance interpretation, nor is it motivated by it. What motivates (QRUE) instead is its natural compatibility with the usual rule for the evolution of the spectral decomposition of mixed states, namely:

$$\hat{U}_t(W_0)\hat{U}_t^{-1} = \hat{U}_t(\Sigma_n w_n(0)\ P_n)\hat{U}_t^{-1} = \Sigma w_n(t)\hat{U}_t\ P_n\hat{U}_t^{-1} = W_t$$

Hence, the ignorance interpretation of mixtures is neither an explicit premise of the insolubility proof, nor is it logically entailed by any of its premises ((TPC), (OOC), (QRUE) and the Schrödinger dynamics).

4.3 The problem of quantum measurement is an idealisation

There is a further question about how appropriate the assumptions made by the insolubility proof are for measurement interactions in general. I have already expressed doubts that (TPC) is an appropriate necessary condition for realistic models of actual measurement interactions. I now want to argue that in the context of the usual tensor-product Hilbert space formalism these assumptions are reasonable. As outside this context the question of a measurement problem does not even arise, the measurement problem is reasonably captured by the insolubility proof.

I will take here an idealisation to be a description of a system which, for the sake of presentation or ease of calculation, involves some assumptions that are known to be false. Thus, what needs to be shown is i) that any false assumptions that may be involved in (TPC), (OOC), (QRUE) or the use of the Schrödinger equation also affect the quantum theory of measurement; and ii) that without those assumptions, the theoretically-based intuition of a measurement problem disappears.

(TPC) is idealised on at least two counts. First, it assumes that whether interactions are measurements is an all-or-nothing affair that depends not on the actual initial state of the system to be measured at a particular time, but on all the possible states that the object may have had in accordance with the theory. This is hardly satisfied by any real measurement we know. For instance, in setting up a localisation measurement of the position of an electron in the laboratory, we do not assume that the device should be able to discern a

he seems to think is logically equivalent to the ignorance interpretation of $\hat{U}\ (W_o \otimes W_a)\ \hat{U}^{-1}$. Stein's condition is indeed necessary and sufficient for (QRUE), and his proof is the closest to the one in Appendix 2. But Stein's condition is not sufficient, only necessary, for (RUE); and hence, in my view, it is not logically equivalent to the ignorance interpretation.

position outside the laboratory walls, even if it is theoretically possible that the particle's position be infinitely far away from us. All real measurement devices are built in accordance with similar assumptions about the *physically* possible, as opposed to the merely theoretically possible, states of the object system, on account of the particular conditions at hand.[9] So real measurement devices do not strictly speaking fulfil (TPC). However, this idealisation has been a part of the quantum theory of measurement from its inception; and it would be very difficult to see how the measurement problem would arise at all in its absence. For if we do not expect quantum theory to completely describe the physically possible initial states of a system, we should hardly expect it to describe completely the physically possible outcomes of a measurement; and that expectation is at the heart of the measurement problem.

The second count of idealisation against (TPC) is that it appears to require measurements to be ideal in the technical sense of correlating one-to-one the initial states of the object system with states of the composite at the end of the interaction. However, many real measurements are not ideal in this sense. Most measurement apparatuses make mistakes, and no matter how much we may try to fine-tune our interaction Hamiltonian, we are likely in reality to depart from perfect correlation.[10] In Section **6** it is argued that, contrary to this appearance, (TPC) is not committed to all measurements being ideal. On the contrary, it is possible to capture a large variety of approximate non-ideal measurements by means of (TPC). In fact (TPC) turns out to be as good a theoretical guide as any for distinguishing those interactions for which a measurement problem can arise from those interactions that it makes no sense even to describe as *measurements*.

Let us now turn to (OOC). This is also idealised since it assumes that the measuring device can only 'point' to the eigenvalue of the pointer position observable which has probability one in the final state at the end of the interaction. The same idealisation is built into the quantum theory of measurement in the form of the (extended e/e link), which was anticipated by Von Neumann's original statement of (basic e/e link). It can of course be relaxed, but only at the expense of introducing new rules for value-ascription into the quantum theory of measurement. In addition, it is clear that without (OOC) there is no measurement problem; for (OOC) captures precisely the intuition that is at the heart of the problem, namely that any quantum measurement

[9] To my knowledge Stein ([1972]) first voiced this concern.

[10] The claim that real measurements are (almost) never ideal in this sense has become common lore in recent philosophy of quantum mechanics, following Albert ([1992]), and Albert and Loewer ([1993]). There are surprisingly few sound arguments offered in favour of this common lore; but it is certainly the case that at least some real measurements (destructive measurements) are not ideal in this technical sense. See Suárez ([1996]) and Del Seta and Suárez ([1999]) for a discussion.

ought to yield an outcome; that is, some outcome or other. Without that intuition, and without the (extended e/e link) to back it up, there is no problem of measurement.

What about (QRUE)? Is it also idealised, and in what respect? (QRUE) assumes that a mixture of pure states of the composite (object + apparatus) evolves into a mixture of the unitarily evolved pure states of the composite. In order to find out whether and how this assumption is idealised, we need to ask the following question: Under what real-life conditions do we expect (QRUE) to fail? We do, without doubt, in cases of environmentally-induced decoherence, for in such cases, the environment induces a non-unitary evolution on the states of the measuring device that is inconsistent with (QRUE). This phenomenon is well known to be ubiquitous in practice; so (QRUE) is indeed strongly idealised. More precisely, (QRUE) assumes that the 'composite' system formed by the quantum objects and the measuring device is isolated from the rest of the universe, which is almost always false in the real world. Yet, notice that the same idealisation is also present in the quantum theory of measurement, which takes the interaction between the object and the apparatus to be unitary, at least prior to the occurrence of an outcome. This assumption has in the past been contested, and is often rightly repudiated in some realistic accounts of measurement, for instance those offered by decoherence and quantum-state diffusion approaches.[11] And although not everyone agrees that the measurement problem is solved completely in these approaches, it is generally agreed that describing the further interaction of the measuring device with its environment takes us closer to a solution of the problem.

Finally, the Schrödinger equation is idealised because it assumes that all quantum systems, not only composite systems involving measuring devices, are closed systems. It assumes that the quantum Hamiltonian can transform pure states into pure states, or mixtures into mixtures, but never a pure state into a mixture or vice-versa. But this again is a pre-requisite for a problem of measurement. For there would be no problem at all if we assumed, as for instance Von Neumann was forced to assume, that at some point in the measurement process a pure-state quantum mechanically evolves into a mixture.

To conclude, the idealising assumptions which pervade the premises of the insolubility proof are concomitant with the quantum theory of measurement itself. The insolubility proof does not trade in a description of the measurement process any more idealised, or any less realistic, than the one offered by the quantum theory of measurement. And it is precisely these idealising

[11] For decoherence approaches to measurement, see e.g. Zurek ([1993]). For the quantum state diffusion approach, see e.g. Percival ([1999]).

assumptions that account for our theoretically-driven intuition that there is something problematic about quantum measurements. Without these idealising assumptions the insolubility proof would be empty; but so would the measurement problem itself.

5 Selections

I have argued that the insolubility proof, in particular Stein's version, succeeds in capturing the essence of the measurement problem. And in one particular respect Stein's version succeeds admirably. His lemma makes explicit the fact that the measurement problem would not arise if the initial states of the system were suitably restricted. For, as Stein writes, the lemma is valid 'if for every nonzero u $\in \nu$, the commutativity condition [...] holds,' where ν is a vector subspace of H, and thus includes all linear combinations of vectors already in ν. In particular, if the superpositions of eigenstates of the object included in ν were discounted, the insolubility proof could not be formulated. The proof does not apply to a space of possible states that excludes arbitrary linear combinations of states already in the space, in other words a space of states that is not a vector subspace. This fact conspicuously points to an appropriate solution to the problem in terms of selections. The rest of this paper is devoted to a discussion of the selections approach.

5.1 Representing dispositional properties

I shall defend the following claim: A selection is an interaction designed to test a particular disposition of a quantum system. Among the dispositional properties I include those responsible for values of position, momentum, spin and angular momentum. In a selection, the pointer position interacts only with the property of the system that is under test.

However, the possibility of selections is not reflected in the formalism of the quantum theory of measurement, which insists on modelling any interaction process by feeding in the full initial quantum state of the object system. On the standard understanding, a quantum state is an array of probability distributions over the eigenvalues of *all* the observables of the system. Thus according to the quantum theory of measurement, any interaction whatsoever with a quantum object is, *ipso facto*, an interaction with all the properties of the object—and hence, on this definition, not a selection.

Something must be added to the formalism to represent selections. We may begin by noting that the quantum state ψ defines a distinct probability distribution for each observable. Hence ψ is an economical representation of all the properties of the system. We may thus wonder if there is a more precise representation, for any quantum system, of each of its properties, individually taken. Suppose that there is a representation W(O) of precisely the property O of a system in state ψ. The least that we would expect W(O) to satisfy is the following consistency condition: W(O) must define exactly the same probability distribution over the eigenvalues of O as does ψ. Thus our desideratum on any more precise representation W(O) of the property O of a system in state ψ amounts to the claim that W(O) be O-indistinguishable from ψ.

It is indeed possible in general to find a more precise representation of each property of a quantum system in state ψ. Consider the following definition of the equivalence class of states relative to a particular observable Q:

$$\textit{Q-equivalence class}: \mathrm{W}' \in [\mathrm{W}]_{\mathrm{Q}} \quad \text{if and only if } \mathrm{W}' \equiv_{\mathrm{Q}} \mathrm{W}.$$

Suppose that O is a (discrete and not maximally degenerate) observable of the system with spectral decomposition given by $\Sigma_n \lambda_n P_n$, where $P_n = P_{[\phi n]} = |\phi_n\rangle\langle\phi_n|$. We can construct the *standard representative* W(O) of the equivalence class $[W]_O$ as follows:

$$\mathrm{W(O)} = \Sigma_n \mathrm{Tr}(\psi \mathrm{P}_n)\mathrm{W}_n, \quad \text{where } \mathrm{W}_n = \mathrm{P}_n/\mathrm{Tr}(\mathrm{P}_n)$$

It is now possible to make the following claim: for a given system in a state ψ, and a given observable O of this system, if ψ belongs to the equivalence class $[W]_O$, then W(O) represents precisely the property O of the system.[12]

A selection of observable O of a specific quantum system in state ψ is then a quantum mechanical interaction (of e.g. the pointer position observable of a measuring device) with the specific property of the system represented by W(O).

5.2 Selections solve the measurement problem

All proposed solutions to the measurement problem so far have tried to tinker formally with the final state of the composite, by replacing the superposition predicted by the Schrödinger equation with an appropriate mixture that will obey OCC. Collapse interpretations do this more or less explicitly, either by introducing an additional dynamics that will yield the appropriate mixture, or (as is the case, for instance, in quantum state diffusion) by replacing the Schrödinger dynamics altogether. No-collapse interpretations do this implicitly. Thus, Everett's 'relative state' is just the mixture that

[12] In Section 7, I ask the question: what kind of properties must these be to be so representable? There is no analogue of this type of representation in classical mechanics. In the classical case, W(O) would simply be the value of a particular dynamical quantity of a system, as extracted from its state; and such extraction is a completely trivial operation. But, as has been emphasised, a quantum state is not to be interpreted *à la* classical mechanics as assignments of actually possessed properties and their values, but rather as a mere assignment of probabilities.

corresponds to a system in an entangled composite when the state of the rest of the universe is a particular eigenstate. The modal interpretation (in its Kochen-Healey-Dieks version) takes the final state of the composite yielded by the Schrödinger equation (the 'dynamical state') to be equivalent to a mixture ('the value state') for the purposes of ascription of values to observables. And Bohmian mechanics advises us to regard every superposition as epistemically reducible to an ignorance interpretable mixture of eigenstates of position.[13]

Now let us suppose that quantum measurements are quantum selections: in a measurement, the pointer position property of the device interacts with only one property of the system, represented by $W_o(O)$. So we must feed this state into the formal representation of the interaction, instead of W_o. From the formal point of view of the quantum theory of measurement, this amounts to 'tinkering' with the initial state. Fine[14] employed this fact to solve the measurement problem: if the initial state that feeds into the Schrödinger equation could be somehow construed as the appropriate mixture over the eigenstates of the object observable, the final state of the composite resulting from Schrödinger evolution would satisfy (TPC) and (OOC).[15]

To see this, let us return to the discussion of measurement interactions, with the definitions of Q-equivalence and the standard representative in mind. A quantum object in state W_o interacts with an apparatus initially in state W_a. We are interested in the property O of the object, represented by the Hermitian operator O with eigenvalues λ_i and eigenvectors ϕ_i. The pointer position observable of the apparatus is represented by the Hermitian operator $I \otimes A$, with eigenvalues μ_{ni} and eigenvectors β_{ni} (corresponding to the eigenvalues μ_n and eigenvectors γ_m of A). The insolubility proof of the measurement problem shows that no unitary interaction can be set up where the probability distribution laid out by W_o over the λ_i eigenvalues of O is matched by that defined by the final state of the composite over the μ_{in} eigenstates of the pointer position observable, *as long as we allow that the initial state of the system may be any arbitrary state*—including, crucially, superpositions of the ϕ_i.

However, we are supposing that in a measurement the pointer position property, represented by $W_a(A)$, interacts only with the property of the system represented by $W_o(O)$.[16] We are then able to model the interaction of a system in a state W_o by a measuring device in state W_a as a selection of the property of

13 For a description of these, and other interpretations of quantum mechanics, see e.g. Albert ([1992]) or Dickson ([1998]).

14 Fine ([1987]), ([1993]).

15 This may suggest that the existence of selections is a *logical* consequence of the insolubility proof, since they are the only interpretation of quantum mechanics that can get around the proof without relinquishing any of the proof's premises. In Section **5.3**, I argue against this suggestion.

16 For the sake of simplicity, and without loss of generality, I assume throughout that $W_a(A) = W_a$.

the system represented by $W_o(O)$, as follows:

$$\begin{aligned} W_o(O) \otimes W_a &\rightarrow \hat{U}_t(W_o(O) \otimes W_a)\hat{U}_t^{-1} \\ &= \hat{U}_t(\Sigma_n Tr(W_o P_n) W_n \otimes \Sigma_m w_m\ P_{[\gamma m]})\hat{U}_t^{-1} \\ &= \Sigma_{nm}\eta_{nm}(t)\hat{U}_t(W_n \otimes P_{[\gamma m]})\hat{U}_t^{-1} \\ &= \Sigma_{nm}\eta_{nm}(t)\hat{U}_t(P_{[\phi n]} \otimes P_{[\gamma m]})\hat{U}_t^{-1} \end{aligned}$$

where $\eta_{nm}(0) = \Sigma_{nm} Tr(W_o P_n) w_m$.

It is now easy to see that as long as this selection satisfies (QRUE), the pointer position observable will take values in the final state of the composite, in accordance with (extended e/e link). For simplicity, consider the ideal, non-disturbing, (QRUE)-obeying interaction U_t:

$$\hat{U}_t(\phi_n \otimes \gamma_m) = \phi_n \otimes \gamma_n$$

This interaction has the following effect:

$$\hat{U}_t(P_n \otimes P_{[\gamma m]})\hat{U}_t^{-1} = \hat{U}_t(P_{[\phi n \otimes \gamma m]})\hat{U}_t^{-1} = P_{[\hat{U}(\phi n \otimes \gamma m)]} = P_{[\phi n \otimes \gamma n]} = P_{[\beta nn]}$$

where β_{nn} is an eigenvector of $(I \otimes A)$ with eigenvalue μ_{nn}. The final state of the composite resulting from this selection is then:

$$W^f_{o+a} = \Sigma_{nm}\eta_{nm}(t)P_{[\beta nn]}$$

This is a mixture over pure states, namely projectors onto the eigenspaces of $(I \otimes A)$. Hence each $P_{[\beta nn]}$ ascribes some value to $(I \otimes A)$ with probability one and, according to (extended e/e link), the pointer position observable takes a value in the state W^f_{o+a}.

5.3 Selections and ignorance

Does the ignorance interpretation play a role in the solution to the measurement problem offered by selections? Perhaps contrary to appearances, it plays no role.

I begin by drawing a distinction between selective interactions and selections. Fine defined selective interactions as unitary interactions with the standard representative of a system that obeyed (TPC) and (QRUE). He was then able to show that such selective interactions solved the measurement problem, for he was able to demonstrate that the final state of the composite resulting from any such selection obeys (OOC). I have defined selections, more generally, to be unitary interactions designed to test a particular property of a system represented by a standard representative. There is no reason in principle why a selection should obey (TPC), or (QRUE). And thus there is no reason in principle why a selection should yield a final state of the composite that satisfies (OOC). Even in the case of selective interactions, which obey (QRUE), there is no entitlement to the ignorance interpretation, for recall from Section **4** that (QRUE) is not sufficient for the ignorance interpretation.

This result has two important consequences. First, it shows why it is a mistake to think of selections in general either as an artefact of the insolubility proof, or as a logical consequence of this proof. Selections turn out to be a more general class of interactions, which include selective interactions as a subset. And although the insolubility proof shows that selections can solve the measurement problem, and thus provides one reason in favour of selections, nothing like a logical demonstration of selections from the premises of the insolubility proof is forthcoming. There is no reason in principle why all selections should obey (QRUE). Even if some selections (selective interactions) obey (QRUE) and get around the insolubility proof, this is hardly the basis for a deduction of this particular set of selections because, as argued in Section **4**, (QRUE) is itself highly idealised and empirically weakly motivated. Additional empirical reasons in favour of the existence of selections must be sought, and that is what I do in Sections **6** and **7** of this paper.

The second consequence requires some preliminary discussion. One may be tempted by the following argument to claim that selections make the mistake of ascribing the wrong state to quantum systems that enter into interaction with measuring devices. Consider the final state of a selective interaction:

$$W^{f}_{o+a} = \Sigma_{nm}\eta_{nm}(t)P_{[\beta nn]}$$

The probabilities η_{nm} are the time-evolutions of the product of the probabilities of the eigenvalues λ_n in the initial state $W_o(O)$ of the object system on the one hand, and of the probabilities of μ_m in the initial state W_a of the apparatus on the other. Now, suppose that in a selection we were required to give the ignorance interpretation to the final state of the composite, and to understand the probabilities η_{nm} as subjective probabilities describing our incomplete knowledge of the 'true' state. And suppose in addition that η_{nm} is constant in time, i.e. $\eta_{nm}(t) = \eta_{nm}(0) = \Sigma_{nm}Tr(W_oP_n)w_m$. This would commit us to understanding $Tr(W_oP_n)$, and w_m as subjective probabilities. It follows that we are required to give the ignorance interpretation to the initial mixed state of the apparatus W_a, and to the standard representative of the object system $W_o(O)$.

It is possible to do so in spite of the argument against the ignorance interpretation of improper mixtures in Section **2** of this paper, because neither W_a nor $W_o(O)$ is in general improper. But giving the ignorance interpretation to $W_o(O)$ raises a puzzle. Recall that $W_o(O)$ is a mixture over W_n states. In giving an ignorance interpretation to it, we are claiming that the true state of the object system at the beginning of the interaction is *really* one of the states W_n with the prescribed probabilities. But the initial state of the system is W_o! This may not even be a mixed state, and it will generally be very different from any of the W_n. Moreover, although the mixture $W_o(O)$ is, by construction, in W_o's equivalence class, neither of the pure states W_n that appear in the decomposition of $W_o(O)$ is.

Considering formally the simple case of a Schrödinger cat-like measurement helps to make the point more clearly. We are invited to consider a two-dimensional observable O with eigenstates ϕ_1 and ϕ_2 and corresponding eigenvalues λ_1 and λ_2 respectively. We are then asked to consider three O-distinguishable states, ϕ_1, ϕ_2 and ϕ_3, where ϕ_3 is the linear combination: $a_1\phi_1 + a_2\phi_2$. Given ϕ_3 and that the spectral decomposition of $O = \lambda_1 P_{[\phi 1]} + \lambda_2 P_{[\phi 2]}$, we can construct the standard representative of $\phi 3$'s O-equivalence class, namely the mixed state: $W_o(O) = |a_1|^2 P_{[\phi 1]} + |a_2|^2 P_{[\phi 2]}$. The argument above entails that in order to solve the Schrödinger cat paradox by means of a selection, we need to give the ignorance interpretation to $W_o(O)$. This amounts to the claim that the system really is in state ϕ_1 or ϕ_2, although we do not know which one exactly. And this contradicts our prior knowledge that the state of the system is ϕ_3 instead. Surely we are not here being asked to entertain the long-discredited ignorance interpretation of superpositions!

The argument is fallacious. It incorrectly assumes that the ignorance interpretation of the final state of the composite is required to solve the measurement problem, and that selections are in the business of providing this by advancing a subjective interpretation of the probabilities. But in light of the previous discussion, i) the measurement problem does not call for the ignorance interpretation of mixtures, proper or improper; ii) the concept of a selection in no way involves the ignorance interpretation; and iii) even those selections that obey (QRUE) and defeat the insolubility proof do not require the ignorance interpretation—in fact they may be inconsistent with it, for the probabilities η_{nm} may evolve in time.

More to the point, the argument misconstrues the selections approach in two different ways. Firstly, on the account of selections developed in this paper, $W_o(O)$ represents not the full state of the system but the state of the O property—taken on its own. Secondly, although $W_o(O)$ is a proper mixture (it does not result from the application of the axiom of reduction to a larger composite, but from the preparation procedure that generated W_o), it is not an ignorance-interpretable one.

I develop these claims in greater detail in Section **7.3**. For now it suffices to point out that to suggest that quantum measurements are quantum selections is not to suggest that there are no systems in superpositions; nor is it to suggest that the actual initial state of a system that is just about to measured is the mixture $W_o(O)$ instead of W_o. That would not agree with experience in cases where W_o is a pure-state superposition: it is always possible to run an interference experiment on the system which can only be modelled correctly by means of the superposition. The suggestion is rather that a system in state W_o has a large number of propensities, each associated with a quantum observable O and represented by $W_o(O)$; and that measurements of the system are selections because measuring devices interact with only one of these propensities at a time.

6 Non-ideal selections

In Section **7.3** it will be argued that the selections approach can be construed as a peculiar variant of modal interpretation. Perhaps the best-known modal interpretation is the so-called Kochen-Healey-Dieks (KHD) interpretation. A widely discussed objection to this interpretation is that it cannot account for non-ideal measurements.[17] In this section, I argue that selections have one important advantage over KHD, namely that they can account for non-ideal measurements naturally. The results of this section thus serve two important purposes. First they provide empirical arguments for the existence of selections, and second they demonstrate one way in which selections are superior to their competitor no-collapse interpretations.

6.1 No-collapse interpretations and non-ideal measurements

In their most elementary version, KHD interpretations ascribe values to the O property of the object system and to the pointer position observable of the measuring device if and only if the final state of the composite is in a biorthonormal decomposition form:

$$|\psi\rangle = \Sigma_i c_i |\phi_i\rangle \otimes |\gamma_i\rangle$$

However, note that this is a small subset of the set of all possible final states:

$$|\psi\rangle = \Sigma_{ij} c_{ij} |\phi_i\rangle \otimes |\gamma_j\rangle$$

in which it is typically not possible to predict perfect correlation between the values of the pointer position observable and the object observable. Ideal interaction Hamiltonians yield final states in the biorthonormal decomposition form; but for the larger class of Hamiltonians that govern non-ideal interactions, the modal interpretation cannot ascribe values in the final state to the pointer position observable.

[17] See Albert ([1992]); Albert and Loewer ([1991], [1993]).

6.2 Exact and approximate measurements

The following terminology will be adopted: an exact measurement may only result from an ideal interaction while an approximate measurement may result from a non-ideal one. Let us begin by characterising these two forms of interaction:

Ideal Interaction: $\Sigma_i c_i \phi i \otimes \gamma_o \rightarrow \Sigma_i d_i \phi_i \otimes \gamma_i$
Non-ideal Interaction: $\Sigma_i c_i \phi i \otimes \gamma_o \rightarrow \Sigma_{ij} d_{ij} \phi_i \otimes \gamma_j$

On the account of measurement adopted in this paper, an ideal interaction is a measurement of O by $I \otimes A$ only if it obeys (TPC): $d_i = c_i$. We may then define an exact measurement as an ideal interaction that obeys (TPC) and correlates possible values of the relevant property of the object system with possible values of the pointer. We may also define the notions of Σ-measurement and approximate measurement as follows:

Σ-*measurement*: A non-ideal interaction is an Σ-measurement if $|d_{ij}|^2 < \in$, if $i \neq j$, where $0 < \in < 1/2$.

Approximate measurement (Shimony [1974]): An Σ-measurement is an approximate measurement if $|d_{ij}|^2 \approx 0$, if $i \neq j$.

In general, ∈-measurements are not proper measurements of the state of the object system. Most ∈-measurements are not (TPC)-obeying, and cannot be used to reliably infer the state of the object system from the experimental outcome. Instead, these measurements generally test for the probabilities of states of the object system given the measurement outcome, and may be used to reliably infer conditional probabilities of states on outcomes.

Approximate measurements are a special kind of Σ-measurements which approximate ideal measurements, and are thus approximately (TPC)-obeying. These are proper measurements of the states of the object, as they allow us to infer the states of the object system to a high approximation.

6.3 Selections for non-ideal interactions

I claim that the selections approach accounts for precisely that subset of Σ-measurements that are proper measurements of the initial state of the object system (as opposed to measurements of conditional probabilities) as well as for all exact and approximate measurements. In other words, selections are not only able to account for non-ideal measurements in general; they also provide a useful wedge to separate very precisely non-ideal interactions from actual measurements.

In the previous sub-section, I showed how any exact measurement may be modelled as an exact selection. Here, I show how selections may model i) Σ-measurements of the initial state of the object system that obey (TPC), and ii) approximate measurements.

A non-ideal selection of a disposition O of a quantum system is a non-ideal interaction of the pointer position property of a measuring device with the O disposition of the system as represented by the standard representative Wo(O):

$$\Sigma_i |c_i|^2 P_{[\phi_i]} \otimes P_{[\gamma_o]} \rightarrow \Sigma_{ij} |d_{ij}|^2 P_{[\phi_i \otimes \gamma_j]}$$

Now it is easy to show that any non-ideal selection obeys (TPC) if and only if it obeys the following general condition:

$$\forall j \ \Sigma_i |d_{ij}|^2 = |c_j|^2$$

But this general condition is also required for Σ-measurements to obey (TPC). We conclude that all Σ-measurements that obey (TPC) can be modelled as non-ideal selections that obey the general condition.

As an illustration, a two dimensional selection that constitutes an Σ-measurement is given by the following three expressions:

1. $(|c_1|^2 P_{[\phi_1]} + |c_2|^2 P_{[\phi_2]}) \otimes P_{[\gamma 0]} \rightarrow$
   $|d_{11}|^2 P_{[\phi_1 \otimes \gamma_1]} + |d_{12}|^2 P_{[\phi_1 \otimes \gamma_2]} + |d_{21}|^2 P_{[\phi_2 \otimes \gamma_1]} + |d_{22}|^2 P_{[\phi_2 \otimes \gamma_2]}$
2. $|d_{11}|^2 + |d_{21}|^2 = |c_1|^2$
3. $|d_{12}|^2 + |d_{22}|^2 = |c_2|^2$

6.4 Approximate selections

Let us now turn to approximate measurements. These may be characterised as selections by means of the general condition:

$$\forall j \ \Sigma_i |d_{ij}|^2 \approx |c_j|^2$$

However, these selections do not strictly obey (TPC), so we may question whether they are measurements at all. We may address the worry by independently developing a fully-fledged account of approximate selections, as follows:[18]

Approximate Selection: An approximate selection of the O property of a system in a pure state ϕ_n is a selection of the O property of a system in the mixed state ρ_n that approximates to ϕ_n, where ρ_n approximates to ϕ_n if $\rho_n = \Sigma_m w^n_m P[\phi_m]$, and $w^n_n \approx 1$.

[18] The account that follows was developed in conversations with Arthur Fine.

An approximate measurement of observable $O = \Sigma_n c_n \phi_n$ on a system in the state $W_o = \Sigma_n c_n \phi_n$ can be modelled as an approximate selection: substitute W_o with the standard representative of its O-equivalence class, namely $W_o(O) = \Sigma_n |c_n|^2 P_{[\phi_n]}$. We may substitute each $P_{[\phi_n]}$ with the mixed state ρ_n, which approximates to it to yield: $\Sigma_n |c_n|^2 \Sigma_m \, w^n_m P_{[\phi_m]}$. We may now run an ideal selection of the O property of this state:

$$\Sigma_{n,m} |c_n|^2 \; w^n_m P_{[\phi_m]} \otimes P_{[\gamma o]} \rightarrow \Sigma_{n,m} |c_n|^2 \; w^n_m P_{[U(\phi_m \otimes \gamma o)]}$$
$$= \Sigma_{n,m} |c_n|^2 \; w^n_m P_{[\phi_m \otimes \gamma_m]}$$

It is easy to check that $\Sigma_{n,m} |c_n|^2 w^n{}_m \, P[U(\phi_m \otimes \gamma_m)] \approx \Sigma_n |c_n|^2 \, P[U(\phi_n \otimes \gamma_n)]$. In words, the state that results from an approximate selection approximates to the final state of the exact measurement given by an ideal (TPC)-obeying selection. This shows that it is legitimate to model approximate measurements by means of approximate selections.

6.5 Implications for ignorance

In Sections **5** and **6** it has been argued that selections do not in general need to obey (TPC) or (QRUE), and in Section **4** I showed that (RUE)—and the ignorance interpretation—might fail even when (QRUE) holds. So selections are not one but two steps away from the ignorance interpretation. And indeed it is easy to show that (RUE) fails in i) non-ideal selections that obey (TPC), such as proper ϵ-measurements; and in ii) non-(TPC)-obeying selections, such as approximate measurements. On the other hand, it is also easy to show that the special kind of ideal selections that do obey (TPC) and (QRUE), such as exact measurements, automatically obey (RUE). The requirement that the probability distribution be matched is, in the case of exact measurements, enough to keep fixed the values of the probabilistic coefficients. This result strengthens the case for the dispensability of (RUE) in the insolubility proofs advanced in Section **4** for it shows that (TPC) and (QRUE) together already do some of the work for which (RUE) has been thought to be necessary.[19]

7 Selective interactions test quantum propensities

In the final section, I turn to interpretational issues. How can we understand selections? And why are measurements selections? I first critically address the

19 Del Seta ([1998]) gives a different argument for the dispensability of (RUE).

answer to these questions given by Fine himself, and then provide my own account in terms of dispositions.

7.1 Equivalence classes as physical 'aspects': a critique

Fine's thought was that some interactions are 'selective' in the sense that they respond only to a certain aspect of the system. For every property of a quantum system originally in a superposition, there is a corresponding mixed state that is probabilistically equivalent (for that property) to the superposition. For instance, a system in a superposition of E-eigenstates $\psi = \Sigma c_i v_i$ is probabilistically indistinguishable, as regards E, from a system in the mixed state $W = \Sigma |c_i|^2 P_{[v_i]}$. An interaction is selective if it has been set up in order to find out about this particular E aspect of the system and no other. In modelling this selective interaction, the mixed state may be used, for the superposition is not a precise enough representation of this and only this aspect of the system. Thus Fine writes:

> The basic proposal, then, is to regard the measurement of an observable E on a system in state ψ as a measurement interaction that selects the aspect of the system corresponding to the probability distribution for E that is determined by state ψ. ([1992], p. 126)

Although I agree with Fine's contention that selections can solve the measurement problem, I disagree with his interpretation of selections as measurements of aspects of physical systems. Fine's interpretation contains counterintuitive elements, and provides a weak motivation for the existence of selections. His suggestion is that we interpret quantum systems in superpositions (regardless of whether individual particles or entangled sets of particles) as made up of smaller subsystems. He writes:

> My exploration starts out from the idea that some interactions are selective. They do not actually involve the whole system, only some physical subsystem. Thus the interaction formalism ought not be applied to the state of the whole system, only a representative of the subsystem engaged in the interaction. ([1987], p. 502)

Fine is here reasoning as follows: a system in a mixture has no 'subsystems'. Hence in interacting with it, a measurement device interacts with the whole system. But, as the system is in a mixture, some outcome will result. By contrast, a system (even if a single particle) in a superposition is made up of several 'subsystems'. In an interference experiment, such as a two-slit experiment, the device interacts with the entire system, or with all the subsytems at once, and this explains why interference terms occur. In a measurement interaction, however, the measuring device will interact only with an individual subsystem. A 'selective interaction' then takes place, and this explains why a precise outcome results with a certain probability.

However, the suggestion that any system in a superposition is made up of several 'subsystems' is counterintuitive from an ontological point of view. For suppose that the system is a single particle. The claim that the particle is composed of further 'subsystems' corresponding to each standard representative is essentially nothing but the claim that the particle is composed of further (smaller?) particles, each of them in a particular quantum state. This brings about a bizarre ontology and leaves us lacking in any explanation for the curious fact that in an interference experiment all the subsystems are interacted with, but not in a measurement.

Suppose, on the other hand, that the initial superposition is a representation of the entangled state of two or more particles. For illustration, consider an EPR pair of particles (1 and 2) in a singlet state of spin 'up' and spin 'down' along the x direction:

$$\psi = (1/\sqrt{2})\ |\mathrm{up_x}\rangle_1\ |\mathrm{down_x}\rangle_2 - (1/\sqrt{2})\ |\mathrm{down_x}\rangle_1\ |\mathrm{up_x}\rangle_2$$

The suggestion that this superposition represents a system made up of further subsystems is even more counterintuitive, for while there is now an unambiguous ontological prescription for individuating these subsystems, it disagrees with Fine's prescription. Fine prescribes the standard representatives for each of the 'subsystems':

$$\begin{aligned} \mathrm{W(x)} &= \tfrac{1}{2}\, \mathrm{P_{[up,down]}(x)} + \tfrac{1}{2}\, \mathrm{P_{[down,up]}(x)}, \\ \mathrm{W(y)} &= \tfrac{1}{2}\, \mathrm{P_{[up,down]}(y)} + \tfrac{1}{2}\, \mathrm{P_{[down,up]}(y)}, \text{ etc.} \end{aligned}$$

However, W(x), W(y) represent distinct properties of the composite system of particles 1 and 2, and cannot be interpreted as states of each of the particles, individually taken. Even if these problems could be solved, it is difficult to see how Fine's prescription may constitute a physical motivation for selections. There is no independent reason why interacting with a 'subsystem' will yield an outcome while interacting with a whole system will not. We certainly do not have an analogue of this in classical physical theories. (In classical mechanics, for instance, we typically assume that a gravitational interaction with a massive object designed to measure its weight will result in an outcome even if the object is constituted by smaller particles. In electrodynamics, measurements of the charge of large conductors give outcomes, even if conductors are made up of smaller, equally charged, parts.) Fine's use of the system/subsystem distinction is *sui generis*, and specifically tailored for quantum mechanics.

I believe that these are definite objections to Fine's interpretation of selections. The basic problem seems to be that Fine's interpretation constitutes a return to an unacceptable understanding of a quantum state as describing a complete set of actually possessed properties of a quantum system. On this understanding, each standard representative must represent a complete set of actually possessed properties of something, which might be (mis)leading Fine into 'subsystem' speech. A better alternative, consistent with the standard understanding of quantum states, is that there is only one system (which may well be a composite), with each standard representative representing a different dispositional property of that system.[20]

7.2 Quantum dispositions

I defend the view that a selection is an interaction of the pointer position observable of a measurement device with a dispositional property of a quantum object. Each dispositional property is displayed under the right test conditions as a chance distribution, represented by the corresponding standard representative. Hence, these properties are *propensities* in the sense of Mellor ([1974]).

On this view, quantum entities do not have further constituent parts or 'subsystems', but they possess dispositional properties.[21] An electron, for instance, possesses a momentum-propensity (let us call it 'momentum'), which is displayed only in the appropriate selection; but the electron typically lacks a specific value of momentum (its wavefunction will rarely be sufficiently peaked in momentum space). The possession of 'momentum' by the electron is 'unconditional', in the terminology of Martin ([1994]) and Mumford ([1998], p. 21): the electron possesses it in the actual world, just like any ordinary object possesses any of its categorical properties. This is perfectly consistent with the electron never in its lifetime acquiring a specific value of momentum, for in the absence of the appropriate selection a propensity may never display itself, or become manifest, just as a fragile glass may never break.

20 Fine is certainly aware of the standard understanding of quantum states. I am at a loss, however, as to how else to interpret the passages quoted above. Perhaps in spirit, if not in letter, Fine's interpretation is closer to a propensity interpretation than it appears.

21 These are similar but not identical to Healey's *dispositional probabilities* (Healey [1989], pp. 54–5). Like Healey, I take it that the manifestation of a quantum disposition is essentially probabilistic, because the application of Born's rule does not typically yield precise values, but precise *probabilities* for values. I go further than Healey in ascribing a property ('momentum', 'position', 'spin', etc) over and above the probability distribution that is manifested; this property is responsible for the distribution in question, and can be ascribed to the system even when the system has no actual value. One could instead seek to ascribe two different properties: 'spin' would then be the property that obtains when and only when a value of spin obtains, while (let us call it) 'spinable' would be the dispositional property that obtains regardless. 'Spinable' would be analogous to the dispositional 'fragility', and 'spin' to the categorical 'breaks': The possession of 'spinable' would explain the occurrence of 'spin', but the dispositional property would not be reducible to the categorical. Some distinction of this kind is desirable for a correct conceptual analysis of dispositional ascriptions but, for the purposes of this paper at least, there is no point in complicating the ontology unnecessarily. It is simpler to work with just one dispositional property ('spin') which obtains always, regardless of whether it is manifested. I want to thank an anonymous referee for pointing out that my view is consistent with Mellor's ([1974]) theory of propensities. Indeed the success of selections in solving the measurement problem can be taken as evidence that the fundamental properties of nature are dispositional, thus vindicating Mellor's thought that it may be dispositions 'all the way down'.

Hence I am adopting a sufficiently robust sense of propensities, which takes them to be possessed by systems even when the test conditions required for their manifestation fail to obtain.[22]

This view of quantum entities as endowed with irreducible propensities,[23] provides us with an extremely natural way to understand selections, and their solution to the measurement problem. A measurement is a (QRUE)-, (OOC)- and (TPC)-obeying selection between the pointer position observable of the measuring device and one of a particle's propensities, an approximately (TPC)-obeying selection in the case of approximate measurements.

Each propensity O of a quantum particle in a superposition W_o is represented by a mixed state $W_o(O)$ with an associated chance distribution that displays the propensity in question. Similarly the measurement device, initially in state W_a, is endowed with a number of propensities, including a 'pointer position propensity', each one represented by a mixed state $W_a(A)$. It is the hallmark of a measurement interaction that the pointer position will interact solely with one particular propensity of the system.

There are partial analogues of this selective character of measurements in both some classical physics measurements, and everyday sensory perception—albeit in both cases the properties interacted with are categorical, or can be suitably conceived as such. For example, measurements of the temperature of a body are typically made by attaching highly sensitive sensors to certain parts of the body. This constitutes an interaction between the measuring device and the temperature of the object; other properties of the body ('mass', 'density', 'electric charge') are not typically thereby interacted with. In our observation of the colour of a table, or our perception of the smoothness of its surface, we interact only with those properties of the table that are responsible for those features: the electromagnetic radiation that the table emits in one case; and the roughness, porosity and texture of its surface, in the other case. The features in question are 'secondary properties' and thus a result in part of these properties of the object and in part of some of the properties of our sensory apparatus. But in either case, the interaction is selective in a way analogous to a quantum selection: in observing the colour, we do not interact with the porosity or texture of the surface; and in detecting the smoothness of the surface, we do not interact with the emitted radiation.

It is then a question of modelling quantum-mechanically the interaction between the measuring device's pointer position and the system's propensity. In a TPC-obeying measurement, the chance distribution displayed by O is

22 In agreement with the theories of Martin ([1994]), Mellor ([1974], [2000]) and Mumford ([1998]).

23 Irreducible at least in the sense, mentioned in footnote 21, that they are not reducible to their manifestations. To the extent that there is an open question about realist hidden variable theories, there is also an open question about whether these propensities are ultimately reducible to some set of, yet unknown, categorical properties.

straightforwardly displayed as the probability distribution of the pointer position observable at the end of the interaction. In an approximate measurement, on the other hand, given the nature of the interaction, the pointer position observable won't display exactly the chance distribution displayed by the system's propensity, but only approximately so. In either case, we can say that the probability distribution defined by $W_o(O)$ is the chance distribution displayed by O.

If we set up a measurement interaction designed to measure a particular propensity, we must take seriously the fact that only the property represented by $W_o(O)$ is interacted with. Otherwise there would be relevant physical facts about the interaction that we would not take into account. The interaction Hamiltonian is the same whatever propensity of the particle we measure. So the standard quantum theory of measurement does not capture whatever genuine physical differences (not merely differences in the experimenter's intentions) obtain between different experimental set-ups designed to measure different dispositions of a quantum system. In that regard, the quantum theory of measurement is incomplete. This is where selections step in: in providing a separate representation of each of a system's propensities, selections allow us to represent relevant physical facts.

Given that all the information about a particle's propensities is encoded in the set of mixed states that represent them, it may seem that the superposition ψ is not needed. ψ has two main functions. First, it is an economical way to represent all the relevant information at once. Instead of writing down a long collection of mixtures to fully characterise a quantum system, I may just write down ψ, from which it is always possible to derive the set of mixtures by means of Fine's algorithm for the standard representative. A second function of ψ, which explains why it is not possible to dispense with it even in principle, is related to the fact that propensities may interact with each other. In quantum mechanics, unusually perhaps, they typically do: testing for a particular disposition of an object precludes us from testing another. No test for the position disposition of a quantum system can be carried out simultaneously with a test for its momentum disposition. This type of information (about which interactions preclude, or constrain, which others) is not encoded in the standard representatives. Only the state ψ of the system contains this type of information. Hence if the experiment is set up to test the interactive character of the dispositions of some quantum particle (such as a two-slit experiment), we must represent the state of the particle by means of the superposition, which fully represents the interference aspect of the physical interaction.[24]

[24] This brings home nicely an added advantage of selections: unlike some interpretations of quantum mechanics, selections do not appeal to the bizarre concept of a self-interacting particle. It is sufficient to accept that each particle's propensity may interact with other

7.3 Selections as a propensity modal interpretation

A comparison with modal interpretations helps to clarify the role of the different state-descriptions within the selections approach. In several respects, the selections approach is a variant of the modal interpretation, albeit one adjusted to make room for propensities, and which obeys (extended e/e link). In van Fraassen's terminology, modal interpretations ascribe two states to a quantum system: the *dynamical* and the *value* state.[25] The dynamical state fully specifies how the system will evolve, whether in isolation or in interaction, determining at all times both the range of possible values of each dynamical quantity, and the probabilities for each value. Typically this role is played by the quantum state ψ, evolving quantum-mechanically in accordance with the Schrödinger equation. The value state, on the other hand, fully specifies which observables of the system have values and what those values are at any time. Modal interpretations often provide us with a rule that allows us to derive the value state from the quantum state under certain circumstances.

On the selections approach, a system in a quantum state ψ has a number of propensities, each represented by a standard representative $W_o(O)$, which describes not values actually possessed by property O of the system, but the chance distribution that displays the system's O propensity when O is measured. So $W_o(O)$ is not the value state, but one element in what we may call the *propensity* state. The propensity state is the set (typically of infinite cardinality) of $W_o(X)$ states, for all observables X of the system. This state represents the non-actualised propensities of the system, and describes the chance distributions that would display them under appropriate measurements, but it cannot be given the ignorance interpretation. We must bear this in mind when applying (extended e/e link).

The dynamical state, on the other hand, is, on the selections approach, a complex entity, composed of the superposition ψ together with the collection of each of the propensity states W(O) of the system. For an isolated system, the evolution of ψ on its own determines the possible values of the system at any later time, and their probabilities: no propensities are actualised. For a system subject to a selection, however, the W(O) are indispensable, for their evolution represents the evolution of the system's O-propensity in a measurement interaction set up to measure O.

propensities, in accordance with the uncertainty relations; hence a system's 'momentum' cannot be manifested, or actualised, simultaneously with its 'position', etc. The claim that different properties of an entity may interact with each other is not controversial if those properties are dispositional. For instance, the fragility of a glass interferes with its capacity to serve as liquid container: these properties cannot be manifested simultaneously, for the manifestation of the former (the breaking of the glass) causes the glass to lose possession of the latter. Hence the selections approach also sheds light on the nature of the uncertainty relations.

25 van Fraassen ([1991], p. 275).

The selections approach agrees with Healey's interactive modal interpretation[26] in emphasising the importance of physical interactions in the ascription of values. So it pays at this stage to describe the situation from the point of view of the composite system (object + apparatus). Suppose the apparatus is set to measure the O propensity of the system: on the selections approach, the propensity state of the composite is given by $W_o(O) \otimes W_a$. The dynamical state is given by the set $\{\psi \otimes W_a, W_o(O) \otimes W_a\}$, but it is irrelevant in the ascription of values. Application of (extended e/e link) to the propensity state gives us the potential 'values' of the combined system, but we always have to bear in mind that these 'values' have not been actualised yet! The only actually possessed values at this stage are those that obtain from the application of (extended e/e link) to the quantum state $\psi \otimes W_a$, so we may want to refer to this as the value state.

At the end of the interaction, the system's dynamical state is given by the unitarily evolved $\{U(\psi \otimes W_a), U(W_o(O) \otimes W_a)U^{-1}\}$. The propensity state is given by $W^f_{o+a} = U(W_o(O) \otimes W_a)U^{-1}$, and so is the value state. Thus on the selections approach, the evolution of the propensity state, which serves as the basis for the ascription of properties to the system, is unitary. But, importantly, the character of the ascription has changed: we start by ascribing potential 'values' of properties, and end up ascribing actual ones. If we concentrate just on what I have here called value state, which records the actually possessed values of the system, we can see that its evolution is not unitary.[27] The non-unitary change in the value state of the composite system precisely represents the event of actualisation, or manifestation, of its $O \otimes A$ propensity. The system goes from having a precise propensity, but no value, corresponding to the $O \otimes A$ property, to having a particular value.

The O-propensity of the object system is displayed as a chance distribution over the possible values of O which, given the TPC-obeying character of any selection, is in turn precisely displayed as the chance distribution over the possible values of $I \otimes A$.[28] But actual values of $I \otimes A$ can only be ascribed at the end of the selection. This is precisely the main advantage of the propensity interpretation of selections: it allows us to legitimately ascribe a property ('spin') to a system, even though the system has not yet gained an actual value of that property.

[26] See Healey ([1989], p. 33).

[27] Does this mean that we need to qualify the claim that selections solve the measurement problem without relinquishing the Schrödinger equation? I do not think so: on any modal interpretation, the Schrödinger equation does not describe the evolution of the specific values of the observables of a particular system, but rather the evolution of the dynamical state. That is also true in the selections approach.

[28] This further illustrates why we should not fall for the ignorance interpretation of $U(W_o(O) \otimes W_a)U^{-1}$. Otherwise, it would not be possible to distinguish between the value and the dynamical states of the composite system at the end of the interaction.

7.4 A comparison with Popper's propensity interpretation

The account that I have been developing takes propensities to be central to the interpretation of quantum mechanics, and to solving the measurement problem. Appeal to quantum propensities is not new, and has a considerable pedigree.[29] Perhaps the best-known proposal in this direction is that due to Karl Popper. In this final section, I would like to briefly distinguish the propensity account of selections from Popper's propensity interpretation of the wave function.[30]

Popper's interpretation defends, among others, the following five theses, roughly described:

1. Propensities are real quantum properties instantiated in nature.
2. Propensities are not monadic properties of isolated quantum systems, but relational properties of quantum entities in experimental set-ups. A one-electron universe would lack any propensities.
3. Quantum theory is essentially a probabilistic theory, in the sense that it is a theory about the probabilities that certain outcomes obtain in certain experimental set-ups.
4. The quantum wavefunction, or state, is a description of a propensity wave over the outcomes of an experimental set-up.
5. Providing an objective interpretation of the probabilities in quantum mechanics in terms of propensities is sufficient to solve the philosophical puzzles concerning quantum mechanics.

The propensity account of selections shares with Popper's interpretation an emphasis on the quantum probability distribution as the basis for the ascription of dispositions. To the extent that a propensity can be defined as probabilistically-quantified dispositional ascription, the account I offer is also a propensity-based one. However, the similarities end there. The account either denies or is non-committal about Popper's theses 1–5.

The propensity account of selections remains neutral about Popper's thesis 1. It is only required that propensities may be ascribed even in the absence of any actual (past, present or future) test. Beyond this requirement, the account neither denies realism about propensity ascriptions nor requires it. In particular, a conditional analysis of probabilistic dispositional properties is acceptable as long as it accommodates this requirement.

Another difference concerns the nature of the quantum propensities themselves. Popper's thesis 2 is false in my propensity account.[31] Although the

[29] Among the founding parents of quantum mechanics, Heisenberg (e.g. [1962]) was particularly keen to understand quantum mechanics in terms of 'potentialities'.

[30] See Popper ([1982]).

[31] Thesis 2 has been convincingly criticised by Peter Milne ([1985]), who shows that it leads to incorrect predictions in the case of the two-slit experiment.

propensities that I take quantum mechanics to ascribe to systems can only be revealed by means of interactions with measuring devices designed to carry out measurements of the appropriate observables, their ascription is fully independent of the existence of such interactions. On my account, an electron in a one-electron universe may be in state ψ, and thus possess all the propensities described by the appropriate standard representatives.

Popper's thesis 3 also turns out false: quantum mechanics is a theory about quantum entities (including, certainly, subatomic particles) and their properties, not about probabilities. It just happens that the properties of quantum entities are dispositional.

The propensity account of selections is not committed to Popper's thesis 4. On this account, the quantum wave function does not directly describe a 'propensity wave'. Instead, the wave function is an economic tool to derive the mixed standard representative states which describe probabilities of outcomes. There is no need for a literal interpretation of the wavefunction as representing a real 'wave'.[32]

The account also denies the spirit, if not the letter, of Popper's thesis 5. Let us leave aside other paradoxical issues of quantum mechanics: merely providing an interpretation of the calculus of probabilities cannot solve the measurement problem, whether objective or subjective. It is necessary instead to work hard on the formal representation of the physics. In particular one has to i) introduce the notion of a selection and represent it formally; ii) provide an interpretation of selections that supports the claim that all measurements are selections; iii) show that the measurement problem only arises in the context of assumptions (TPC), (QRUE), (OOC) and the Schrödinger equation, and iv) show that there is no measurement problem for those selections that obey (TPC), (QRUE), (OOC) and the Schrödinger equation. It has been my intention in this paper to provide substantial arguments for all these four claims, thus providing a background against which these claims can be most fruitfully analysed and debated.

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[32] In addition, Neal Grossman ([1972]) showed that Popper's interpretation fails to distinguish appropriately between mixtures and superpositions—a problem that does not affect the propensity account of selections.

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Appendix 1: The interaction formalism

The quantum theory of measurement provides the tensor-product space formalism provided by the quantum theory of measurement to represent the interaction between an object system and a measuring device. Given two Hilbert spaces, H_1 and H_2, we can always form the tensor-product Hilbert space $H_{1+2} = H_1 \otimes H_2$, with dim $(H_1 \otimes H_2) = \dim (H_1) \times \dim (H_2)$. If $\{v_i\}$ is a basis for H_1 and $\{w_j\}$ is a basis for H_2, then $\{v_i \otimes w_j\}$ is a basis for H_{1+2}. Similarly if A is an observable defined on H_1 with eigenvectors $\{v_i\}$ and eigenvalues $\{a_i\}$, and B an observable on H_2 with eigenvectors $\{w_i\}$ and eigenvalues $\{b_j\}$ then $A \otimes B$ is an observable on H_{1+2} with eigenvectors $\{v_i \otimes w_j\}$, and corresponding eigenvalues $\{a_i b_j\}$.

Consider two systems S_1 and S_2. If the state of $S_{1'}$s is W_1 on H_1, and the state of $S_{2'}$s is W_2 on H_2, we can represent the state of the combined system S_{1+2} as the statistical operator $W_{1+2} = W_1 \otimes W_2$ acting on the tensor-product Hilbert space H_{1+2}. If either W_1, W_2 is a mixture, then W_{1+2} is also a mixture. If, on the other hand, *both* W_1, W_2 are pure states then W_{1+2} is pure. Suppose that $W_1 = P_{[\psi]}$, and $W_2 = P_{[\phi]}$, where $\psi = \Sigma_i c_i v_i$ and $\phi = \Sigma_j d_j w_j$. Then $W_{1+2} = \Sigma_{i,j} c_i\ d_j v_i \otimes w_j$, which is a superposition of eigenstates of $A \otimes B$ in H_{1+2}. More specifically, if S_1, S_2 are in eigenstates of A, B, the combined system S_{1+2} is in an eigenstate of $A \otimes B$. If $W_1 = v_i$ and $W_2 = w_j$, then $W_{1+2} = v_i \otimes w_j$, a so-called *product state*.

For an arbitrary (pure or mixed) state W_{1+2} of the combined system, and arbitrary observable $A \otimes B$ the Generalised Born Rule applies. The probability that $A \otimes B$ takes a particular $a_i b_j$ value is given by:

$$\mathrm{Prob}_{W1+2}(A \otimes B = a_i b_j) = \mathrm{Tr}(W_{1+2} P_{ij})$$

The expectation value of the 'total' $A \otimes B$ observable in state W_{1+2} is:

$$\mathrm{Exp}_{W1+2}(A \otimes B) = \mathrm{Tr}((A \otimes B) W_{1+2})$$

We will sometimes be given the state W_{1+2} of a composite system, and then asked to figure out what the reduced states W_1, W_2 of the separated subsystems must be. Given a couple of observables A and B on H_1, H_2, there are some relatively straightforward identifications that help to work out the reduced states, namely:

$$Tr((A \otimes I)W_{1+2}) = Tr(AW_1)$$
$$Tr((I \otimes B)W_{1+2}) = Tr(BW_2) \qquad (*)$$

where I is the identity observable. This amounts to the demand that the probability distribution over the eigenspaces of observable A(B) defined by the reduced state $W_1(W_2)$ be the same as that laid out over the eigenspaces of $A \otimes I$ $(I \otimes B)$ by the composite state W_{1+2}, thus effectively ensuring that the mere choice of description (either in the larger or smaller Hilbert space) of a subsystem in a larger composite system has no measurable consequences as regards the monadic properties of the individual subsystems.

Appendix 2: The insolubility proof

Consider three O-distinguishable initial states of the object system:

$$P_{[\phi_1]}, P_{[\phi_2]}, P_{[\phi_3]}$$

where ϕ_1, ϕ_2 are eigenvectors of O with eigenvalues λ_1, λ_2, and ϕ_3 is a non-trivial superposition $\phi_3 = a_1\phi_1 + a_2\phi_2$.

Set up a Schrödinger interaction, in accordance with (QRUE) and (OOC):

$$\hat{U}_t(P_{[\phi_i]} \otimes W_a)\hat{U}_t^{-1} = \Sigma w_n P_{[\beta_{ni}]}$$

whereby (QRUE) $\beta_{ni} = \hat{U}_t\ (\phi_i \otimes \gamma_n)$ and whereby (OOC) $\forall n \forall i = 1, 2, 3$: $\hat{I} \otimes \hat{A}\ (\beta_{ni}) = \mu_{ni}\ \beta_{ni}$.

By the linearity of $\hat{U}_t$:

$$\hat{U}_t(\phi_3 \otimes \gamma_n) = a_1\hat{U}_t(\phi_1 \otimes \gamma_n) + a_2\hat{U}_t(\phi_2 \otimes \gamma_n)$$

Hence $\beta_{n3} = a_1\beta_{n1} + a_2\ \beta_{n2}$.

Now we can calculate:

(A)
$$\begin{aligned}\hat{I} \otimes \hat{A}(\beta_{n3}) &= \hat{I} \otimes \hat{A}(a_1\beta_{n1} + a_2\beta_{n2}) \\ &= a_1(\hat{I} \otimes \hat{A})\beta_{n1} + a_2(\hat{I} \otimes \hat{A})\beta_{n2} \\ &= a_1\mu_{n1}\beta_{n1} + a_2\mu_{n2}\beta_{n2},\end{aligned}$$

and

(B)
$$\begin{aligned}\hat{I} \otimes \hat{A}(\beta_{n3}) &= \mu_{n3}\beta_{n3} \\ &= \mu_{n3}(a_1\beta_{n1} + a_2\beta_{n2}) \\ &= a_1\mu_{n3}\beta_{n1} + a_2\mu_{n3}\beta_{n2}.\end{aligned}$$

However, (A) and (B) are equal if and only if $\mu_{n1} = \mu_{n2} = \mu_{n3}$, in which case β_{n1}, β_{n2}, β_{n3} are not $(\hat{I} \otimes \hat{A})$ – distinguishable. Thus (TPC) fails for this choice of initial states of the system. QED.

Appendix 3: Stein's lemma and its implications

Stein's Lemma: If Q and R are bounded linear operators on the Hilbert spaces H_2 and $H_1 \otimes H_2$ respectively; if ν is a vector subspace of H_1; and if for every non-zero $u \in \nu$ the commutativity condition $S_u = (P_u \otimes Q)\ R = R(P_u \otimes Q)$ holds; then there is a uniquely determined bounded linear operator T on H_2 such that:

$$S_u = P_u \otimes T, \quad \text{for every nonzero } u \in \nu.$$

Application to the Measurement Problem: Take Q to be the initial state of the apparatus, i.e. $Q = W_a$, and R to be the inverse time-evolved pointer position observable, i.e. $R = U^{-1}\ (I \otimes A)U$. It is straightforward that $U(P_u \otimes Q)\ U^{-1}$ commutes with $(I \otimes A)$ if and only if $P_u \otimes Q$ commutes with R. In addition, according to the results in Section **4** of the paper, this commutativity condition holds if and only if (QRUE) and (OOC) hold for $P_u \otimes W_a$.

Stein's lemma then shows that there is a uniquely determined bounded linear operator T on H_2 such that $S_u = P_u \otimes T$. However the quantum statistical algorithm predicts that the expectation of the pointer position observable when the system is in the initial state $P_u \otimes W_a$ is: $\mathrm{Tr}\ (U(P_u \otimes W_a)U^{-1}\ I \otimes A) = \mathrm{Tr}\ (P_u \otimes T)$, which is equal to Tr (T) because the trace of P_u is one. So the expectation of the pointer position observable is independent of the initial state of the system, and no measurement at all has been carried out.

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