Opinion

'If' and the problems of conditional reasoning

Ruth M.J. Byrne[1] and P.N. Johnson-Laird[2]

[1] School of Psychology and Institute of Neuroscience, Lloyd Building, Trinity College Dublin, University of Dublin, Dublin 2, Ireland
[2] Psychology Department, Green Hall, Princeton University, Princeton, New Jersey, 08544, USA

'If' is a puzzle. No consensus has existed about its meaning for over two thousand years. Here, we show how the main psychological theories deal with the seven crucial problems that it raises. These competing explanations treat 'if' as though it was a term in a formal logic, or as eliciting the construction of a mental model of the world, or as an instruction to suppose that a proposition holds. The solution to 'if' would be a major step towards understanding how people reason, and towards implementing a computer program that can reason in a human way. We argue that the mental model theory is closer to resolving the puzzle of 'if' than its competitors.

Introduction

When John F. Kennedy addressed the Irish parliament, he said:

> If this nation had achieved its present political and economic stature a century or so ago, my great-grandfather might never have left New Ross ..

Like Kennedy, you can think hypothetically. You understand 'if' and draw inferences from the conditional assertions that it makes, and neither task seems difficult. Yet for millennia, no agreement has existed about the meaning of conditionals or about their logical properties. Now, following intensive experimental and theoretical efforts, cognitive scientists seem closer to discovering the truth about 'if'. Here, we outline seven problems that any theory of conditionals must solve, and we outline the solutions offered by each of three main sorts of theory. They are theories based on formal logic, on mental models of the world and on the idea that 'if' invites you to make a supposition. We argue that the mental model theory is most likely to solve the puzzle of conditionals: it explains the factors that affect reasoning from conditionals, what they mean, their truth or falsity, their counterfactual use, their denials and their probabilities.

Problem 1: why are some conditional inferences difficult?

Psychologists study four standard inferences exemplified in Table 1. Consider this one:

If he spoke then she laughed.
He spoke.
What follows?

The conclusion is obvious: she laughed. This affirmative inference is 'valid', that is, its conclusion holds in every possibility in which its premises hold [1], and so if its premises are true then its conclusion is too.

By contrast, this negative inference is harder:

If he spoke then she laughed.
She didn't laugh.
What follows?

A typical error is to respond, nothing follows [2], but a valid conclusion is: he didn't speak. This difference in difficulty is very robust and all theories of conditionals explain it.

Theories based on formal logic, as Box 1 shows, explain the difference: there is a formal rule of inference for the affirmative inference, but not for the negative one. The mental model theory, as Box 2 shows, explains the difference: mental models represent the possibility needed for the affirmative inference, but not the possibility needed for the negative inference. And the theory based on suppositions, as Box 3 shows, explains the difference: the supposition represents the case needed for the affirmative inference, but not the case needed for the negative inference.

Problem 2: what do conditionals mean?

Conditionals mean many things [3]. Suppose someone tells you: if the shelf collapsed then someone put a heavy object on it. You infer that the event in the then-clause occurred 'before' the event in the if-clause. Other conditionals convey the opposite temporal order. Formal rule theories (Box 1) take the meaning of conditionals to be implicit in the inferences that follow from them. However, their rules yield too few sorts of inference (e.g. they do not account for the inference above). One solution is to posit more than one formal logic in the mind, but now the problem is to match a conditional to its appropriate logic. This problem itself calls for reasoning [4].

You can envisage the possibilities to which a basic conditional refers, for example: if he laughed then he remembered. Children tend to list just a single possibility, *he laughed and remembered*; young adolescents list this possibility and a second one, *he didn't laugh and didn't remember*; and older adolescents and adults list these

Corresponding author: Byrne, R.M.J. (rmbyrne@tcd.ie).

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Table 1. Examples of the four standard conditional inferences (and their technical names)

	Affirmative	Negative
Valid	If he spoke then she laughed. He spoke. Therefore, she laughed. *Modus ponens*	If he spoke then she laughed. She didn't laugh. Therefore, he didn't speak. *Modus tollens*
Invalid	If he spoke then she laughed. She laughed. Therefore, he spoke. *Affirmation of the consequent*	If he spoke then she laughed. He didn't speak. Therefore, she didn't laugh. *Denial of the antecedent*

possibilities and a third one: *he didn't laugh but remembered*. A better predictor of performance than age, however, is the capacity of working memory [5]. Where do these sets of possibilities come from? The answer must be: from a representation of the meaning of conditionals. Only mental models provide such a representation, and they allow that knowledge can modulate possibilities to introduce, say, temporal relations [3].

Problem 3: in what cases are conditionals true?

In what cases is this conditional true: if there is a circle then there is a triangle? The obvious answer is when there is both a circle and a triangle. Likewise, it is false when there is a circle but no triangle. But, is it true or false when there is not a circle? Many people say: neither, the case is irrelevant [3,6]. This pattern of evaluations is known as the 'defective truth table', and it does not square with the cases that individuals spontaneously list as possible (see the previous section). To evaluate the truth or falsity of an assertion depends first on understanding the assertion, and so the model theory takes the listing of possibilities as fundamental. The suppositional theory explains the defective truth table as a result of making a supposition of the if-clause, and then, depending on whether or not the then-clause is true, evaluating the conditional as true or false [6,7]. But, when the if-clause itself is false, it is as though the conditional is rendered void – an idea developed by the logician Frank Ramsey, although he was referring to belief in a conditional rather than to its truth or falsity (Ref. [8] p.155).

A decisive objection can be made against the defective truth table. Consider the inference:

If Obama wins the US election in 2012 then the Republicans will have lost.

Therefore, Obama wins the US election in 2012.

Box 1. Formal rule theories

This sort of theory postulates that the mind contains a set of formal rules of inference akin to a logical calculus [39,41]. The rules apply once the *logical form* of propositions has been matched to them. A simple rule of inference is *modus ponens*:

If A then B.
A.
Therefore, B.

It immediately delivers the conclusion to the valid affirmative inference in Table 1. But, a more complex rule, *reductio ad absurdum,* is needed for the difficult negative inference [39]. It begins with a supposition, and if, together with other premises, the supposition leads to a contradiction of the form, *A and not A*, then one can deny the supposition. The premises of the negative inference are:

(i) If he spoke then she laughed.
(ii) She didn't laugh.

The proof proceeds:

(iii) A supposition: He spoke (the first step of the reductio rule).
(iv) ∴ she laughed (the modus ponens rule).
(v) ∴ she laughed and she didn't laugh (a rule for 'and' conjoins 4 and 2).
(vi) ∴ He didn't speak (the final step of the reductio rule).

Formal rule theories postulate that inference is a search for a proof of a conclusion, and that if no such proof is discovered then the inference is invalid. Errors arise from the misinterpretation of premises, a misapplication of a rule or even the use of a faulty rule. A recent theory proposes that humans are equipped with more than one logic [4], which seems plausible, but which exacerbates the problem of recovering the logical form of everyday conditionals.

Variant theories use rules with a specific content [42], or in the form of an innate module for reasoning about a specific topic, such as detecting cheating [43]. These theories have almost always been applied to a single problem in conditional reasoning: the selection of evidence pertinent to the truth or falsity of a conditional. This task has generated a large literature, but we will say no more about it because it yields inconsistent data (cf. Refs. [30,44]).

Box 2. The theory of mental models

The theory postulates that reasoning depends on envisaging possibilities. Individuals construct models of the possibilities to which the premises refer [2,3], and they draw conclusions that hold in these possibilities. They reject a conclusion if they find a counterexample, that is, a possibility in which the premises hold, but the conclusion does not [45–47].

The theory gives a 'dual process' account of reasoning [2]. One process is intuitive. Given the conditional: 'If she played a game then she didn't play music,' you construct a single explicit *mental* model representing the possibility in which both clauses hold. You realize that the if-clause might be false, but you do not represent these possibilities explicitly. The mental model suffices for the easy affirmative inference in Table 1. The other process is deliberative and relies on working memory. You envisage the three possibilities to which the conditional refers, one at a time, in *fully explicit* models, which we abbreviate as:

game	not music
not game	music
not game	not music

These possibilities suffice for the difficult negative inference in Table 1.

The meaning and reference of a conditional's clauses, its context, and general knowledge can *modulate* the meaning of 'if' [3]. It can lead individuals to flesh out mental models into fully explicit models. It can also block the construction of models. Consider the conditional: if she played a game then she didn't play soccer. Unlike the similar conditional above, it is consistent with only two possibilities:

game	not soccer
not game	not soccer

because your knowledge that soccer is a game prevents you from envisaging the possibility:

not game	soccer

This blocking of possibilities yields ten distinct interpretations of conditionals [3]. And there are still other interpretations. Modulation, as we show in the main text, can introduce temporal and other relations between events.

The theory makes no use of logical form (Box 1), and it implies that inferences of the same 'grammatical' form differ in whether or not they are valid. Likewise, the interpretation of conditionals cannot depend solely on whether their clauses are true or false because models represent possibilities, and modulation might have introduced a temporal relation between the clauses. Hence, the theory implies that 'conditionals are not truth functional' (Ref. [3] p.673), notwithstanding claims to the contrary [6,30,48].

Box 3. Suppositional theories and the probability of conditionals

Tailor-made for conditionals alone, and based on Ramsey's test (see main text), these theories posit that conditionals elicit suppositional thinking (i.e. you suppose that the if-clause is true, and think about the consequences) [6,7,40,48]. One consequence might be that you reach a contradiction, and so accordingly you can reject your supposition. This idea is common to formal rule theories, which invoke the rule of *reductio ad absurdum*, and to the model theory, which relies on models to make the same inference [3]. But, according to suppositional theories, you also use suppositions to estimate the probability of a conditional [6,7] (c.f. Ref. [30]). The probability of the conditional:

> If the dice came down with an even number uppermost then it came down 6.

is the 'conditional probability' of a 6 given that the dice landed with an even number. Given a fair dice, this probability is 1/3. The same idea applies even when you use indirect evidence to estimate the probability that, say, if Obama's policies work then he'll be re-elected. Some authors have gone even further, and argued that the meaning of a conditional in an everyday context *is* a conditional probability [31].

One suppositional theory postulates a 'singularity principle' that you make only a single mental simulation [6]. Given, say, the conditional, 'If Paul wins the lottery then he'll buy a Ferrari', you simulate Paul winning the lottery and assess the believability of him buying the Ferrari. You represent the strength of your belief as a

The premise is undoubtedly true, and so according to the defective truth table the inference is valid because the only way the premise can be true is if its if-clause is true too [9]. The inference is absurd; and so too is the defective truth table. The judgments of irrelevance that it reflects occur because individuals tend to understand:

> In what cases is it true that if *that* happens then *this* happens?

as meaning:

> If *that* happens then in what circumstances is it true that *this* happens?

They mistakenly evaluate, not the truth of the conditional, but the truth of its consequent given that its antecedent holds.

Problem 4: why is reasoning with *if and only if* easier?

The phrase, 'if and only if', seems like something that only a lawyer or logician would say, but consider this inference [3]:

> If and *only if* he spoke then she laughed.
> She didn't laugh.
> What follows?

You should find it easier to infer that he did not speak than the negative inference that you made in the first section. Similarly, consider this inference:

> He yawned only if she did.
> She didn't yawn.
> What follows?

Again, you should find it easy to infer that he did not yawn. What explains these two robust results [3,10,11]?

The model theory predicts them in the following way [3]. You normally think only about the possibility in which both clauses of a conditional hold. *Only if* makes you also think of the negative possibility in which neither clause holds (e.g. he didn't yawn and she didn't yawn). And it is this possibility that you need to make the negative inference. Formal rule theories and the suppositional theory offer no immediate explanation of the phenomena. They might postulate that both conditionals have the logical form of: *If A then B, and if not-B then not-A*. The second clause enables the negative inference to be made at once. But, how individuals recover this logical form – or indeed any logical form – remains a mystery. Meta-analyses and the close fitting of the three theories to data have shown that the model theory provides a better account of the standard inferences in Table 1 than the other two theories [12–14].

Problem 5: what makes counterfactuals special?

Kennedy's conditional at the start of this article refers to a possibility that did not actually occur – Ireland was still struggling for independence a century before he spoke. Such 'counterfactual' conditionals are common in everyday discourse [15–18], but they differ in meaning and inferential consequences from the conditionals that we have discussed so far. Counterfactuals, as the model theory postulates, refer to two sorts of possibility [3,18]. One possibility is factual: Ireland had not achieved its present stature then and Kennedy's great-grandfather left. The other possibility is counterfactual. It is an alternative to reality: the nation achieved its present stature a century ago and Kennedy's great-grandfather did not leave. These two possibilities yield a difference in meaning from regular conditionals, which refer only to real possibilities. The regular conditional, 'If Shakespeare didn't write *The Tempest* then someone else did', is true, whereas the counterfactual, 'If Shakespeare hadn't written *The Tempest* then someone else would have', is debatable – an obvious alternative is that no-one wrote the play [17–20]. You tend to represent both the factual and the counterfactual possibilities [21–23], and hence a regular conditional primes you to read a description of only the factual possibility, whereas the counterfactual primes you to read descriptions of both the factual and the counterfactual possibilities [24,25]. One inferential consequence that the model theory predicts is that individuals make the valid negative inference in Table 1 more readily with a counterfactual than with a regular conditional [18,26]. Neither formal rule theories nor the suppositional theory offer any obvious account of this result.

Problem 6: what denies a conditional?

Individuals have no clear idea about how to 'negate' an assertion, but they do know how to deny one. Some individuals deny a conditional such as, 'if he spoke then she laughed', by asserting: *if he spoke then she didn't laugh*; and others deny it categorically: *he spoke and she didn't laugh* [27]. But, which denial is correct? The answer is clear for a general claim, such as: if any man speaks then a woman laughs. Its denial is at least one man speaks and a woman does not laugh (i.e. one counterexample suffices to show that the conditional is false [9,28]). Likewise, this case is the only one that is not possible given the meaning of the conditional (Box 2).

So, why do some individuals deny one conditional with another conditional, and why do some theorists [27,29] think that this denial is correct? The answer reflects a general principle, which we introduced in discussing the truth of conditionals (see earlier). Individuals make life simpler for themselves. They understand:

What denies if *that* happens then *this* happens?

as meaning:

If *that* happens then what denies that *this* happens?

They mistakenly envisage, not the denial of the conditional, but the denial of its consequent given its antecedent [9].

Problem 7: what is the probability of a conditional?

The principle to which we have just alluded also explains the answer that individuals give to the following sort of question:

What is the probability that if the nickel is heads then the dime is heads?

They interpret the question to mean:

If the nickel is heads then what's the probability that the dime is heads?

They estimate, not the probability of the conditional, but the conditional probability of its consequent given its antecedent [9]. Formal rule theories do not seem to have addressed this question, but several probabilistic theories of reasoning have postulated that the probability of a conditional is, and *should be*, the aforesaid conditional probability [30,31]. It is also a consequence of the suppositional theory [6,7].

No-one doubts that some people some of the time make this estimate [29,32–34]. But, is it correct? Here is one argument to the contrary, which hinges on this inference:

The nickel came down heads or the dime came down heads, or both did.

Therefore, if the nickel didn't come down heads then the dime did.

Box 4. Outstanding questions

- In what circumstances do people express their thoughts by using 'if' rather than another connective?
- Do people ever recover the logical form of assertions? If so, how do they do it?
- How do individuals come up with *numerical* estimates when they infer the probabilities of conditionals in an intuitive way from non-numerical information?
- What factors guide people in the development and completion of 'what if' thoughts?
- Do individuals simulate an unfolding temporal sequence of events when they understand conditionals referring to sequences of events?

The conclusion holds in exactly the same three possibilities in which the premise holds, and so the two assertions should have the same probability. Suppose the two coins are tossed fairly. The first assertion holds in three out of the four equally likely outcomes, and so it has a probability of 3/4. But, the conclusion's conditional probability that the dime came down heads given that the nickel didn't come down heads is only 1/2. Something has gone wrong. And the only viable diagnosis is that it is erroneous to assume that the conditional probability corresponds to the probability of the conditional.

No-one doubts that some people some of the time make other estimates of the probability of a conditional [29,32–34]. The model theory offers an explanation. As in other forms of reasoning, different individuals adopt different strategies [35,36]. One strategy contrasts the single explicit mental model of a conditional with the alternative in which the conditional is false (Box 2). This 'equiprobable' strategy is analogous to the suppositional theory [37,38]. It too yields a probability of 1/2. A second strategy treats the conditional as though it were a conjunction, again reflecting its single explicit mental model. It yields a probability of 1/4. And a third strategy sums the probabilities of all three possibilities in which the conditional holds. It yields a probability of 3/4, which matches the required probability of the premise in the inference above.

Conclusions

We have described seven problems about conditionals that concern their meanings, their truth or falsity, their counterfactual use, their denials, their probabilities and the factors that affect reasoning from them. They are not the only problems, of course (Box 4), but any adequate theory of conditionals has to solve them. Table 2 summarizes the solutions of the three main theories.

Table 2. The three theories' solutions to the seven problems of conditionals

Theories	Formal rules	Mental models	Suppositions
Problems			
1. Why are some conditional inferences difficult?	+	+	+
2. What do conditionals mean?	–	+	?
3. In what cases are conditionals true?	?	+	–
4. Why is reasoning with *if and only if* easier?	–	+	–
5. What makes counterfactuals special?	–	+	?
6. What denies a conditional?	?	+	?
7. What is the probability of a conditional?	?	+	?

+: the theory offers a putative solution to the problem;.
–: the theory has difficulty in solving the problem;.
?: the theory requires further development to solve the problem.

Formal rule theories focus on the proofs of given conclusions. Their biggest problem is the lack of any comprehensive account of the logical form of assertions: no algorithm exists for deriving it. As a consequence, these theories fail to deal adequately with the meanings of conditionals and their alternative patterns of inference. The theories are not designed to address several of our problems.

The suppositional theory focuses on beliefs and subjective estimates of the probabilities of conditionals. Its biggest problems derive from its emphasis on possibilities in which the if-clause of a conditional holds. This emphasis leads to dubious claims about the defective truth table, denials of conditionals, and estimates of their probability. The theory resembles the model theory in its use of representations of possibilities. It also resembles formal rule theories because its proponents argue that logical form has a role in reasoning. Unlike the formal rule theory [39], however, it has yet to be implemented computationally, and so this aspect of the theory might change.

The model theory focuses on the possibilities to which assertions refer, and uses them to explain conditional reasoning. Recently, we circulated a question to a hundred reasoning researchers: what is wrong with the model theory? Only a small number of them identified putative failures of the theory, but the majority of those who did, criticized its account of conditionals (and thereby in part prompted this article). One objection *is* a conundrum for the theory: individuals with a working memory of a high capacity are more likely to treat conditionals as though they had a defective truth table than as referring to three possibilities [40]. Indeed, no single theory has a monopoly on the truth. Yet, the model theory of conditionals and its computational implementations explains the problems that we have considered here.

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References

1 Jeffrey, R.C. (1981) *Formal Logic: Its Scope and Limits.* McGraw-Hill
2 Johnson-Laird, P.N. (2006) *How We Reason.* Oxford University Press
3 Johnson-Laird, P.N. and Byrne, R.M.J. (2002) Conditionals: a theory of meaning, inference, and pragmatics. *Psychol. Rev.* 109, 646–678
4 Stenning, K. and Van Lambalgen, M. (2008) *Human Reasoning and Cognitive Science.* MIT Press
5 Barrouillet, P. *et al.* (2000) Conditional reasoning by mental models: chronometric and developmental evidence. *Cognit.* 75, 237–266
6 Evans, J.St.B.T. and Over, D.E. (2004) *If.* Oxford University Press
7 Evans, J.St.B.T. (2007) *Hypothetical Thinking.* Psychology Press
8 Ramsey, F.P. (1990) General propositions and causality. In *Foundations: Essays in Philosophy, Logic, Mathematics and Economics* (Mellor, D.H., ed.), pp. 145–163, (Original work published 1929.), Humanities Press
9 Johnson-Laird, P.N. *et al.* (2009) The mental model theory of conditionals: A reply to Guy Politzer. *Topoi* 28, 75–80
10 Santamaria, C. and Espino, O. (2002) Conditionals and directionality: on the meaning of if versus only if. *Q. J. Exp. Psychol.* 55A, 41–57
11 Grosset, N. and Barrouillet, P. (2003) On the nature of mental models of conditional: The case of *If. If then and Only if. Think. Reason.* 9, 289–306
12 Oberauer, K. (2006) Reasoning with conditionals: A test of formal models of four theories. *Cognit. Psychol.* 53, 238–283
13 Espino, O. *et al.* People think about what is true for conditionals, not what is false: Only true possibilities prime the comprehension of "if". *Q. J. Exp. Psychol.*(in press)
14 Schroyens, W. and Schaeken, W. (2003) A critique of Oaksford, Chater and Larkin's (2000) conditional probability model of conditional reasoning. *J. Exp. Psychol. Learn. Mem. Cogn.* 29, 140–149
15 Mandel, D.R. *et al.*, eds (2005) *The Psychology of Counterfactual Thinking*, Routledge
16 Markman, K. *et al.* (2008) *The Handbook of Imagination and Mental Simulation.* Psychology Press
17 Byrne, R.M.J. (2002) Mental models and counterfactual thoughts about what might have been. *Trends Cogn. Sci.* 6, 426–431
18 Byrne, R.M.J. (2005) *The Rational Imagination: How People Create Alternatives to Reality.* MIT Press
19 Bennett, J. (2003) *A Philosophical Guide to Conditionals.* Oxford University Press
20 Williamson, T. (2007) Philosophical knowledge and knowledge of counterfactuals. In *Philosophical Knowledge: Its Possibility and Scope* (Beyer, C. and Burri, A., eds), pp. 89–124, Rodopi
21 Ferguson, H.J. and Sanford, A.J. (2008) Anomalies in real and counterfactual worlds: an eye-movement investigation. *J. Mem. Lang.* 58, 609–626
22 Thompson, V.A. and Byrne, R.M.J. (2002) Reasoning about things that didn't happen. *J. Exp. Psychol. Learn. Mem. Cogn.* 28, 1154–1170
23 Quelhas, A.C. and Byrne, R.M.J. (2003) Reasoning with deontic and counterfactual conditionals. *Think. Reason.* 9, 43–66
24 Santamaria, C. *et al.* (2005) Counterfactuals and semifactuals prime alternative possibilities. *J. Exp. Psychol. Learn. Mem. Cogn.* 31, 1149–1154
25 De Vega, M. *et al.* (2007) Canceling updating in the comprehension of counterfactuals embedded in narrative. *Mem. Cognit.* 35, 1410–1421
26 Byrne, R.M.J. (2007) Precis of The Rational Imagination: How People Create Alternatives to Reality. *Behav. Brain Sci.* 30, 439–480
27 Handley, S.J. *et al.* (2006) The negated conditional: Litmus test for the suppositional conditional? *J. Exp. Psychol. Learn. Mem. Cogn.* 13, 378–395
28 Byrne, R.M.J. and Johnson-Laird, P.N. (2009) Conditionals and possibilities. In *The Psychology of Conditionals* (Oaksford, M., ed.), Oxford University Press
29 Politzer, G. (2007) Reasoning with conditionals. *Topoi* 26, 76–95
30 Oaksford, M. and Chater, N. (2007) *Bayesian Rationality: The Probabilistic Approach to Human Reasoning.* Oxford University Press
31 Pfeifer, N. and Kleiter, G.D. (2005) Towards a mental probability logic. *Psychol. Belg.* 45, 71–99
32 Evans, J.St.B.T. *et al.* (2003) Conditionals and conditional probability. *J. Exp. Psychol. Learn. Mem. Cogn.* 29, 321–335
33 Oberauer, K. and Wilhelm, W. (2003) The meaning(s) of conditionals: Conditional probability, mental models, and personal utilities. *J. Exp. Psychol. Learn. Mem. Cogn.* 29, 680–693
34 Over, D.E. *et al.* (2007) The probability of causal conditionals. *Cognit. Psychol.* 54, 62–97
35 Van der Henst, J-B. *et al.* (2002) Strategies in sentential Reasoning. *Cogn. Sci.* 26, 425–468
36 Lee, N.Y.L. *et al.* (2008) The psychological problem of Sudoku. *Think. Reason.* 14, 342–364
37 Girotto, V. and Johnson-Laird, P.N. (2004) The probability of conditionals. *Psychologia* 47, 207–225
38 Girotto, V. and Johnson-Laird, P.N. (2008) Conditionals and probabilities. In *The Psychology of Conditionals* (Oaksford, M., ed.), Oxford University Press
39 Rips, L.J. (1994) *The Psychology of Proof.* MIT Press
40 Evans, J.St.B.T. *et al.* (2007) Thinking about conditionals: A study of individual differences. *Mem. Cognit.* 35, 1772–1784
41 O'Brien, D. (2009) Human reasoning requires a mental logic. *Behav. Brain Sci.* 32, 96–97
42 Cheng, P.C. and Holyoak, K.J. (2008) Pragmatic reasoning schemas. In *Reasoning: studies of human inference and its foundations* (Adler, J.E. and Rips, L.J., eds), pp. 827–842, Cambridge University Press
43 Cosmides, L. *et al.* (2005) Detecting cheaters. *Trends Cogn. Sci.* 9, 505–506

44 Oberauer, K. *et al.* (2004) Logical reasoning and probabilities: A comprehensive test of Oaksford and Chater (2001). *Psychon. Bull. Rev.* 11, 521–527
45 Johnson-Laird, P.N. and Hasson, U. (2003) Counterexamples in sentential reasoning. *Mem. Cognit.* 31, 1105–1113
46 Schroyens, W. *et al.* (2003) In search of counterexamples: Deductive rationality in human reasoning. *Quart. J. Exp. Psychol.* 56 (A), 1129–1145
47 Verschueren, N. *et al.* (2005) Everyday conditional reasoning: A working memory-dependent tradeoff between counterexample and likelihood use. *Mem. Cognit.* 33, 107–119
48 Evans, J.S.T.B.T. *et al.* (2005) Supposition, extensionality and conditionals: A critique of the mental model theory of Johnson-Laird and Byrne (2002). *Psychol. Rev.* 112, 1042–1052